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MATH 250/GRACEY

WORKSHEET/2.3

Use the following "short-cut" rules to evaluate the derivative of the following functions. Identify $f(x)$ and $g(x)$. Fully simplify your result, writing as a single rational expression with positive exponents, when applicable.

See below

Product rule: $\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$

Extension of the product rule:

$$\frac{d}{dx}\left[f(x)g(x)h(x)\right]=f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$$

Quotient rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{\left[g(x)\right]^2}$

Remaining trigonometric functions: $\frac{d}{dx}[\tan x]=\sec^2 x$ $\frac{d}{dx}[\cot x]=-\csc^2 x$

$\frac{d}{dx}[\sec x]=\sec x\tan x$ $\frac{d}{dx}[\csc x]=-\csc x\cot x$

1. $\frac{\partial}{\partial x}y=\frac{\partial}{\partial x}\left(x\cos x+\tan x\right)$

$y'=\frac{\partial}{\partial x}\left(x\cos x\right)+\frac{\partial}{\partial x}(\tan x)$

$y'=\left[\frac{\partial}{\partial x}(x)\right]\left[\cos x\right]+\left[x\right]\left[\frac{\partial}{\partial x}\cos x\right]+\sec^2 x$

$y'=1\cos x+x(-\sin x)+\sec^2 x$

$\boxed{y'=\cos x-x\sin x+\sec^2 x}$

2. $h(x)=\frac{\sqrt{x}}{x^2-1}$

$h'(x)=\frac{\left[\frac{\partial}{\partial x}x^{1/2}\right]\left[x^2-1\right]-\left[x^{1/2}\right]\left[\frac{\partial}{\partial x}(x^2-1)\right]}{(x^2-1)^2}$

$2x^{3/2}$

$h'(x)=\frac{\frac{1}{2}x^{-1/2}(x^2-1)-x^{1/2}(2x)}{(x^2-1)^2}$

$$h'(x) = \frac{\frac{1}{2}x^{-1/2}\left[(x^2-1) - 4x^2\right]}{(x^2-1)^2}$$

$$\boxed{h'(x) = \frac{-3x^2-1}{2x^{1/2}(x^2-1)^2} = -\frac{3x^2+1}{2x^{1/2}(x^2-1)^2}}$$

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$$h(x) = \cot x = \frac{\cos x}{\sin x}$$

$$h'(x) = \frac{\left[\frac{\partial}{\partial x}(\cos x)\right][\sin x] - [\cos x]\left[\frac{\partial}{\partial x}\sin x\right]}{[\sin x]^2}$$

$$h'(x) = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$h'(x) = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$h'(x) = -\frac{1}{\sin^2 x}$$

$$h'(x) = -\csc^2 x$$

# Product rule

$$\frac{\partial}{\partial x}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Consider $h(x) = (2x+3)(1-x)$

$$h(x) = 2x - 2x^2 + 3 - 3x$$

$$h(x) = -2x^2 - x + 3$$

$$h'(x) = -4x - 1 + 0$$

$$h'(x) = -4x - 1$$

Now let's do same problem using the product rule for derivatives.

$$h(x) = (2x+3)(1-x)$$

$f(x) = 2x+3$ $\quad$ $g(x) = 1-x$

$f'(x) = 2$ $\quad$ $g'(x) = -1$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(x) = 2(1-x)$$

$$\frac{\frac{1}{2} + 2}{3} = \frac{\frac{1+4}{2}}{3}$$

3. $h(t)=\dfrac{\sqrt[4]{t}}{2\sqrt[4]{t}-1}$

4. $y=\left(x^2-1\right)\sec x\csc x$

5. $y=\left(2x^3-5\right)^2$

6. Find the equation of the tangent line at $t=4$ for the function $s(t)=\dfrac{t}{1-t}$.