When you finish your homework you should be able to…

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

Consider the function $f(x)=\dfrac{x^3-8}{x-2}$

Let's graph the function:

Now let's examine the limit of the function as x approaches 2 using a table. This is written as $\lim_{x\to 2} f(x)$, or specifically $\lim_{x\to 2}\frac{x^3-8}{x-2}$ for our function.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Complete the table and use the result to estimate the limit.

1. $\lim_{x\to -3}\frac{\sqrt{1-x}-2}{x+3}$

x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$f(x)$						

2. $\lim\limits_{x \to 0} \cos \dfrac{1}{x}$

x	$1/\pi$	$1/2\pi$	$1/3\pi$	$1/4\pi$	$1/5\pi$	$1/6\pi$
$f(x)$						

3. Consider the function $f(x) = \dfrac{x}{x-3}$

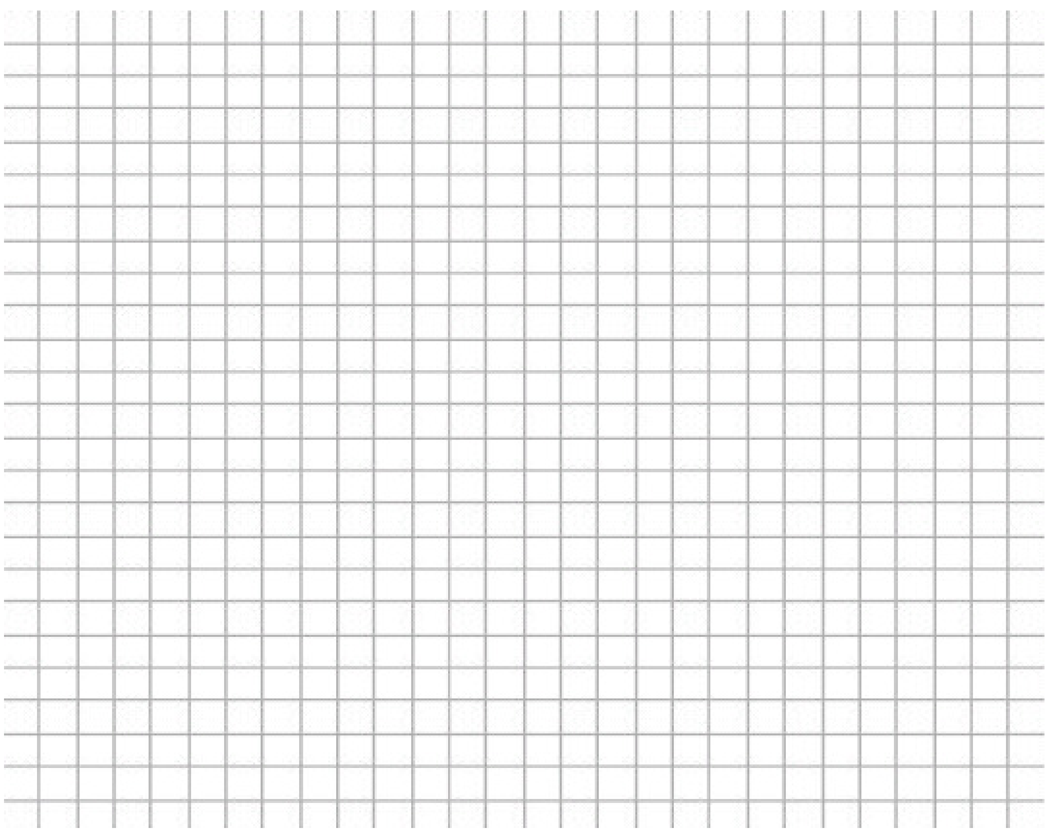
Let's graph the function:

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

What observations would you make about the behavior of this function at the asymptote?

4. Consider the function $g(x)=\begin{cases} \sin x, & x \le 0 \\ 1-\cos x, & 0 \le x \le \pi \\ \cos x, & x > \pi \end{cases}$

Let's graph the function:



Now let's identify the values of c for which the $\lim_{x \to c} g(x)$ exists.

DEFINITION OF LIMIT

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a **real number**. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each small positive number epsilon, denoted ε, there exists a small positive number delta, denoted ∂, such that if

$$0 < |x - c| < \partial, \text{ then } |f(x) - L| < \varepsilon.$$

5. Find the limit *L*. Then find $\partial > 0$ such that $\left|f(x) - L\right| < 0.01$ whenever $\left|x - c\right| < \partial$.

$$\lim_{x \to 5} \left(x^2 + 4\right)$$

6. Find the limit *L*. Then use the epsilon-delta definition to prove that the limit is *L*.

$$\lim_{x \to 4} \sqrt{x}$$