MATH 250/GRACEY

WORKSHEET/3.1-3.2

Definition of Extrema

Let f be defined on an open interval I containing c.

1. $f(c)$ is the **minimum of** f **on** I if $f(c) \le f(x)$ for all x in I.

2. $f(c)$ is the **maximum of** f **on** I if $f(c) \ge f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values**, or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum** on the interval.

Relative Extrema

If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f, or you can say that f has a **relative maximum at** $(c, f(c))$.

2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f, or you can say that f has a **relative minimum at** $(c, f(c))$.

Critical Numbers

Let f be defined at c. If $f'(c) = 0$, or if f is not differentiable at c, then c is a **critical number** of f.

1. Find any critical numbers of the function.

a. $f(x) = 2x^2 - 8x + 5$

b. $g(x) = \sin^2 x - \sin 2x, \ [0, 2\pi)$

c. $s(t) = \dfrac{3t}{t^2 + 2}$

2. Find the value of the derivative at the extremum $(0,4)$, for the function

$$f(x)=\cos\frac{\pi x}{2}.$$

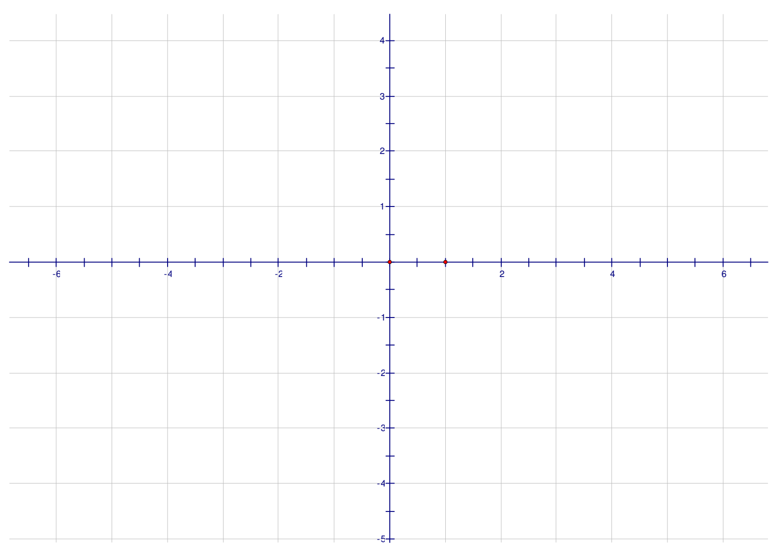
GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a,b]$, use the following steps.

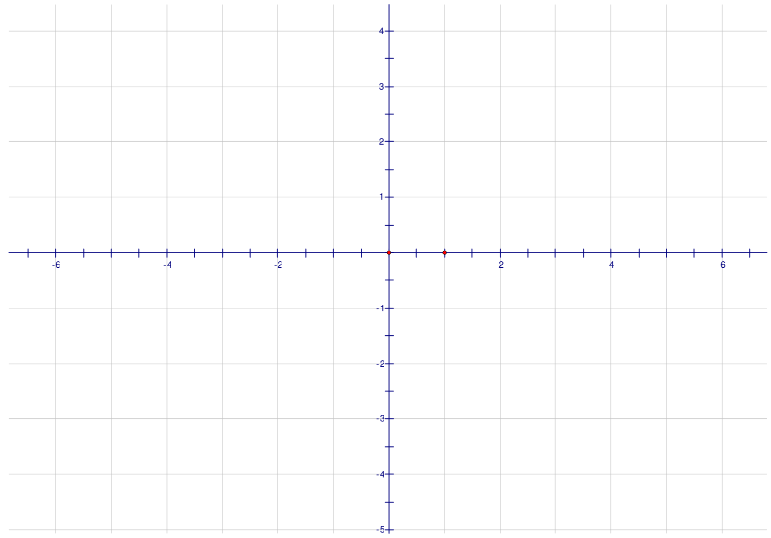
1. Find the critical numbers of f in (a,b).
2. Evaluate f at each critical number in (a,b).
3. Evaluate f at each endpoint of $[a,b]$.
4. The least of these numbers is the minimum. The greatest is the maximum.

3. Locate the absolute extrema of the function on the closed interval.

a. $f(x)=x^3-12x,\ [0,4]$



b. $g(x) = \frac{x}{x-2}, \ [3,5]$



4. A retailer has determined that the cost *C* of ordering and storing *x* units of a product is $C = 2x + \frac{300,000}{x}, \ 1 \le x \le 300.$ The delivery truck can bring at most 300 units per order.

   a. Find the order size that will minimize cost.

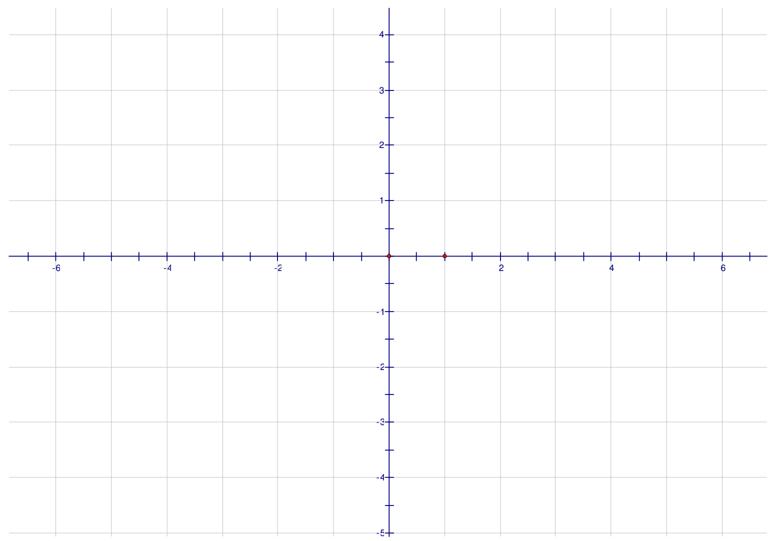
   b. Could the cost be decreased if the truck were replaced with one that could bring at most 400 units? Explain.

ROLLES THEOREM

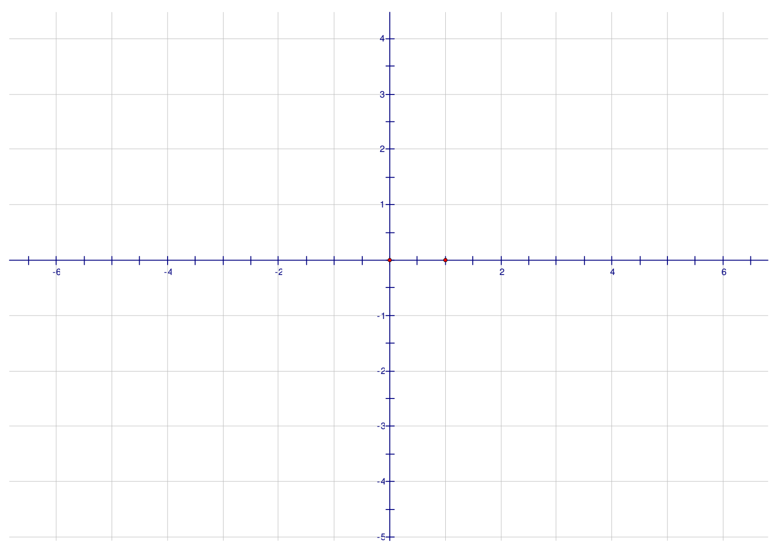
> Let f be continuous on a closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.

5. Determine whether Rolle's Theorem can be applied to *f* on the closed interval $[a,b]$. If Rolle's Theorem can be applied, find all values of *c* in the open interval (a,b) such that $f'(c)=0$.

   a. $f(x)=\cos 2x,\ \left[-\frac{\pi}{12},\frac{\pi}{6}\right]$



   b. $f(x)=\frac{x^2-1}{x},\ [-1,1]$

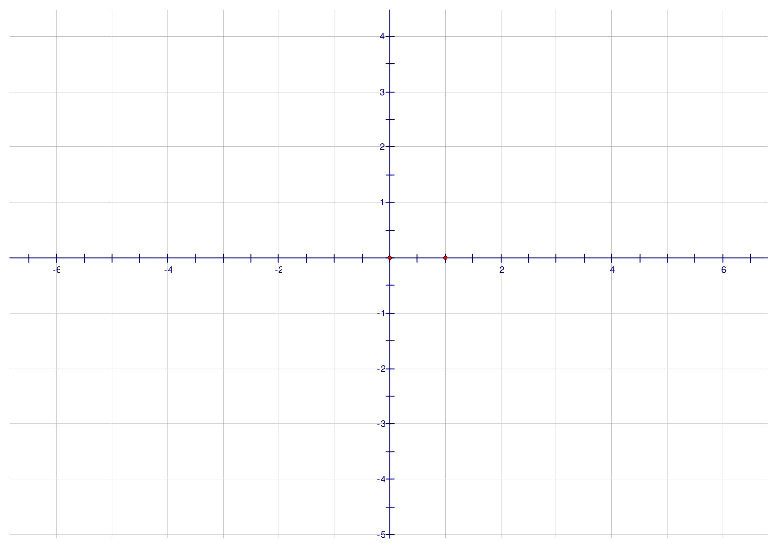


THE MEAN VALUE THEOREM

> If f is continuous on a closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists at least one number c in (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

6. Determine whether the Mean Value theorem (MVT) can be applied to f on the closed interval $[a,b]$. If the MVT can be applied, find all values of c such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

   a. $f(x)=\dfrac{x+1}{x}, \left[\dfrac{1}{2},2\right]$



7. A plane begins its takeoff at 2:00PM on a 2500-mile flight. The plane arrives at its destination at 7:30PM. Explain why there are at least two times during the flight when the speed of the plane is 400 miles per hour.