MATH 250/GRACEY NAME________________________

WORKSHEET/3.3-3.4

Definitions of Increasing and Decreasing Functions

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b).

1. If $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on $[a,b]$.
2. If $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on $[a,b]$.
3. If $f'(x) = 0$ for all $x \in (a,b)$, then f is constant on $[a,b]$.

Guidelines for Finding Intervals on which a Function is Increasing or Decreasing

Let f be continuous on the interval (a,b). To find the open intervals on which f is increasing or decreasing, use the following steps.

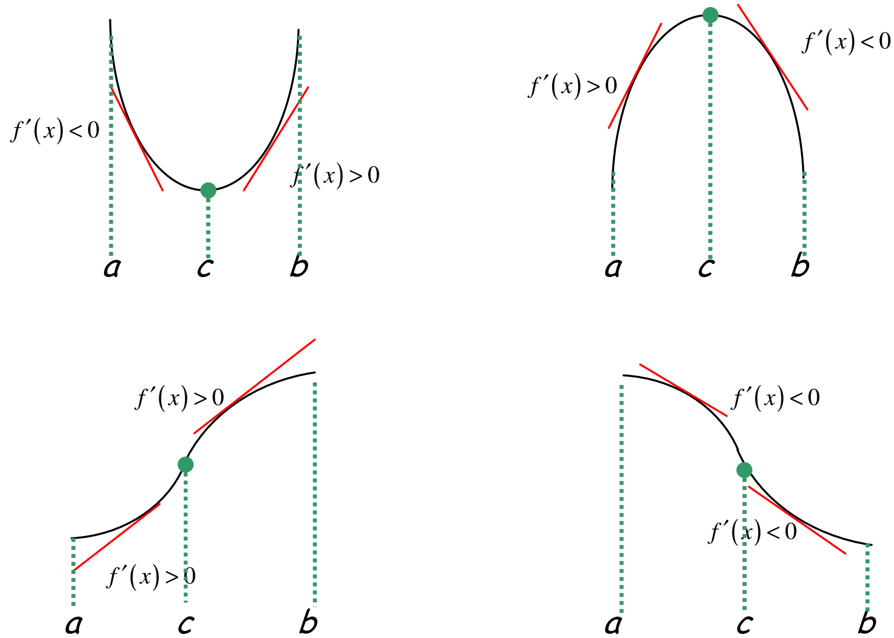
1. Locate the critical numbers of f in (a,b) and use these numbers to determine the test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Use the previous theorem to dtermine whether f is increasing or decreasing on each interval.

These guidelines are also valid if the interval (a,b) is replaced by an interval of the form $(-\infty,b)$, (a,∞), or $(-\infty,\infty)$.

Theorem: The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum or relative maximum.



1. Consider the function $g(x)=\dfrac{x^2-3x-4}{x-2}$.

   a. Find the critical numbers of g (if any):

   b. Find the open intervals on which g is increasing (if any):

   c. Find the open intervals on which g is decreasing (if any):

   d. Apply the First Derivative Test to identify all relative extrema:

Theorem: Second Derivative Test

Let f be a function such that $f'(c)=0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c)>0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c)<0$, then f has a relative maximum at $(c, f(c))$.
3. If $f''(c)=0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.

Theorem: Test for Concavity

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x)>0$ for all x in I, then the graph of f is concave upward in I.
2. If $f''(x)<0$ for all x in I, then the graph of f is concave downward in I.

Definition of a Point of Inflection

Let f be a function that is continuous on an open interval and let c be a point in the interval. If the graph of f has a tangent line at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f if the concavity of f changes from upward to downward (or downward to upward) at the point.

Theorem: Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c)=0$ or f'' does not exist at $x=c$.

2. Consider the function $g(x)=2\sin x+\cos 2x,\ [0,2\pi)$.

   a. Find the critical numbers of g (if any):

   b. Find all relative extrema:

   c. Find the points of inflection and discuss the concavity of g :

3. The graph below is the graph of the derivative $f\,'$ of some unknown function f .

a. Identify the interval(s) on which f is increasing.

b. Identify the interval(s) on which f is decreasing.

c. Estimate the value(s) of x at which f has a relative maximum or minimum.

d. Identify the intervals on which f is concave upwards.

e. Identify the intervals on which f is concave downwards.