

• SIGMA NOTATION

The sum of  $n$  terms  $a_1, \dots, a_n$  is written as  $\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$  where  $i$  is the **index of summation**,  $a_i$  is the  **$i$ th term** of the sum, and the **upper and lower bounds of summation** are  $n$  and 1.

1. Find the sum.

$$\sum_{i=1}^5 i^2$$

Summation Properties

1. 
$$\sum_{i=1}^n ka_i = k \sum_{i=1}^n a_i$$

2. 
$$\sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

Theorem: Summation Formulas

1. 
$$\sum_{i=1}^n c = cn$$

2. 
$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

3. 
$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

4. 
$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

2. Evaluate the following sums.

a.  $\sum_{i=1}^n (3i - i^2)$

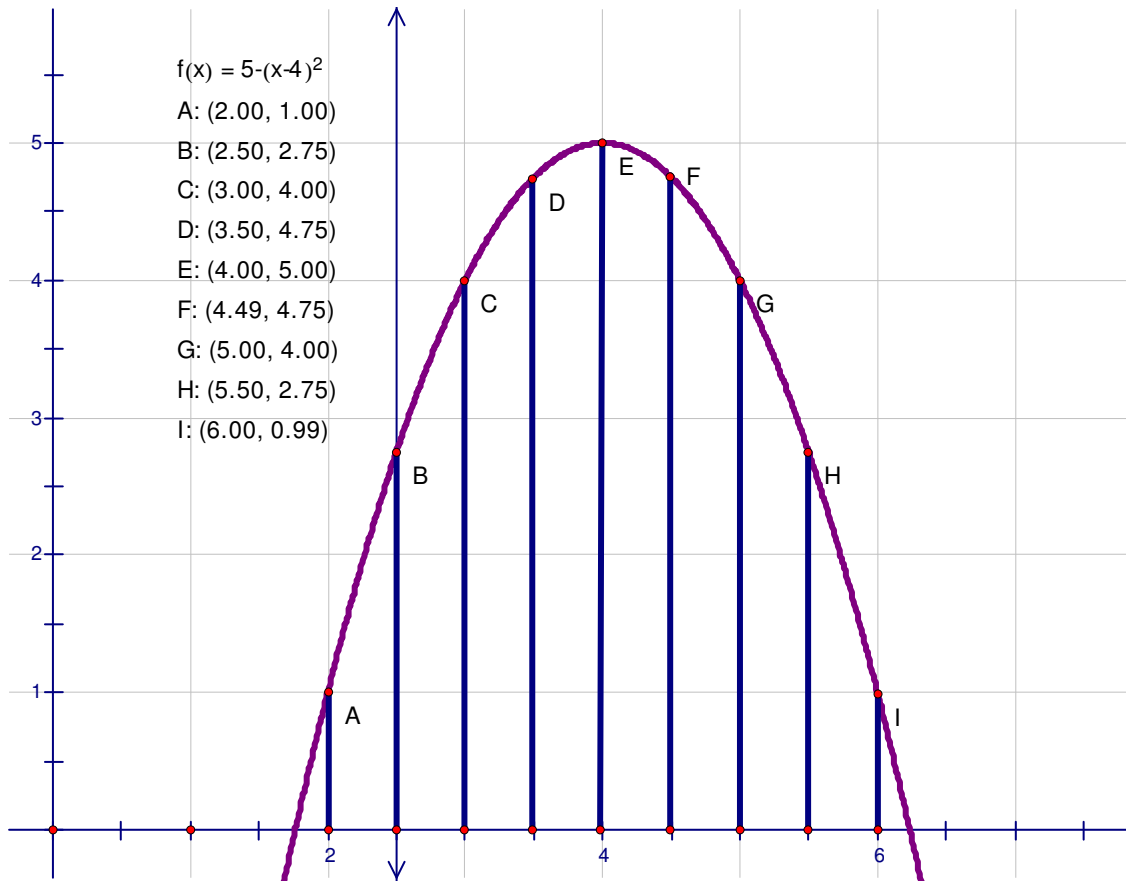
b.  $\sum_{i=1}^n (6i + 4i^3)$

**Theorem: Limits of the Lower and Upper Sums**

Let  $f$  be continuous and nonnegative on the interval  $[a, b]$ . The limits as  $n \rightarrow \infty$  of both the lower and upper sums exist and are equal to each other. That is,

$$\begin{aligned}\lim_{n \rightarrow \infty} s(n) &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(m_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(M_i) \Delta x \\ &= \lim_{n \rightarrow \infty} S(n)\end{aligned}$$

where  $\Delta x = \frac{b-a}{n}$  and  $f(m_i)$  and  $f(M_i)$  are the minimum and maximum values of  $f$  on the subinterval.



3. Find the upper and lower sums from  $x = 2$  to  $x = 6$ .

### Definition of an Area in the Plane

Let  $f$  be continuous and nonnegative on the interval  $[a, b]$ . The area of the region bounded by the graph of  $f$ , the  $x$ -axis, and the vertical lines  $x = a$  and  $x = b$  is

$$\text{Area} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x, \quad x_{i-1} \leq c_i \leq x_i$$

$$\text{where } \Delta x = \frac{b-a}{n},$$

right endpoint:  $c_i = a + i\Delta x$ , left endpoint:  $c_i = a + (i-1)\Delta x$

4. Find the area of the region bounded by the graph  $f(x) = x^3$ , the  $x$ -axis, and the vertical lines  $x = 0$  and  $x = 1$ .