MATH 250/GRACEY NAME________________________

WORKSHEET/4.3

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4+2n^3+n^2}{4}$$

1. Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x_i$ over the region bounded by the graphs of $f(x)=\sqrt[3]{x},\ y=0,\ x=0,\ x=1$. Hint: Let $c_i = i^3/n^3$ and recall that the width of each interval is $\Delta x_i = \frac{i^3}{n^3} - \frac{(i-1)^3}{n^3}$.

2. Evaluate the definite integral by the limit definition.

$$\int_1^6 \left(2x^2+1\right)dx$$

3. Write the limit as a definite integral on the interval $[a,b]$ where c_i is any point on the ith interval.

a. $\lim\limits_{\|\Delta\|\to 0} \sum\limits_{i=1}^{n} (8c_i + 15)\Delta x_i, \quad [2,6]$

b. $\lim\limits_{\|\Delta\|\to 0} \sum\limits_{i=1}^{n} 5c_i\sqrt{c_i^2+2}\Delta x_i, \quad [0,12]$

4. Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

a. $\int_{-2}^{6} 6dx$

b. $\int_{-4}^{4} \sqrt{16 - x^2}\,dx$

5. Given $\int_{0}^{3} f(x)dx = 4$ and $\int_{3}^{6} f(x)dx = -1$, evaluate

a. $\int_{0}^{6} f(x)dx$

b. $\int_{6}^{3} f(x)dx$

c. $\int_{3}^{3} f(x)dx$

d. $\int_{3}^{6} f(x)dx$