MATH 250/GRACEY NAME______________________

WORKSHEET/4.3

Theorem: The Fundamental Theorem of Calculus

> If a function f is continuous on the closed interval $[a,b]$ and F is an antiderivative of f on the interval $[a,b]$, then
>
> $$\int_a^b f(x)\,dx = F(b) - F(a)$$

Guidelines for Using the Fundamental Theorem of Calculus

1. Provided you can find an antiderivative of f, you now have a way to evaluate a definite integral without having to use the limit of a sum.
2. When applying the Fundamental Theorem of Calculus, the following notation is convenient:

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b$$

$$= F(b) - F(a)$$

3. It is not necessary to include a constant of integration C in the antiderivative because

$$\int_a^b f(x)\,dx = F(x) + C\Big]_a^b$$

$$= \left[F(b) + C\right] - \left[F(a) + C\right]$$

$$= F(b) - F(a)$$

1. Evaluate the definite integral.

a. $\int_{-2}^{6} 6dx$

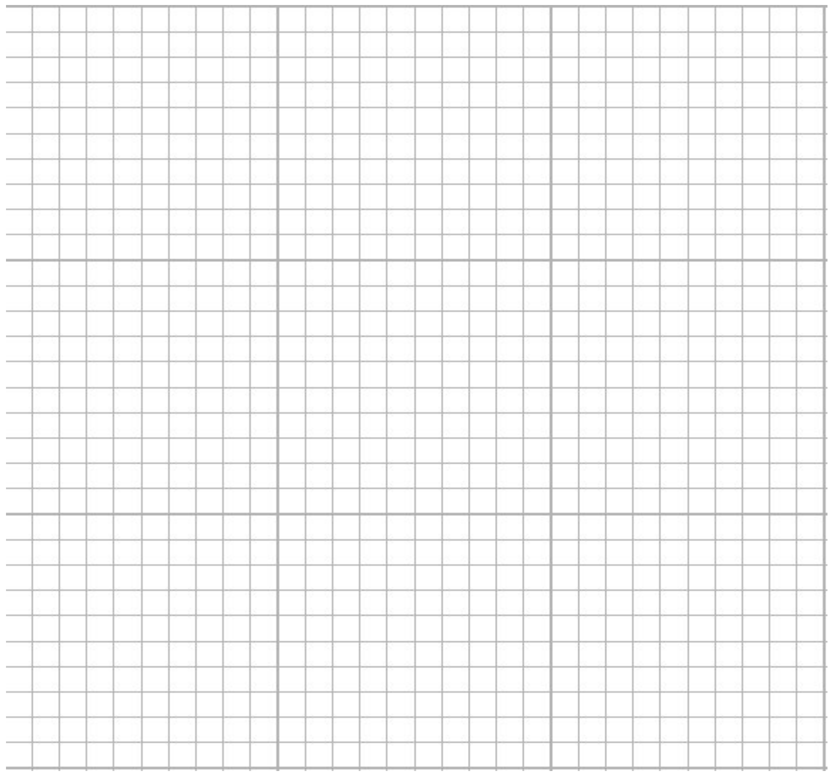
b. $\int_{1}^{6} \left(2x^2 + 1\right)dx$

c. $\int_{0}^{2} (2-t)\sqrt{t}\,dt$

d. $\int_{1}^{4} (2v+5)^3 dv$

2. Find the area of the region bounded by the graphs of the equations.

$$y = 1 + \sqrt[3]{x}, \quad x = 0, \quad x = 8, \quad y = 0.$$



THE MEAN VALUE THEOREM FOR INTEGRALS

If f is continuous on the closed interval $[a,b]$, then there exists a number c in the closed interval $[a,b]$ such that

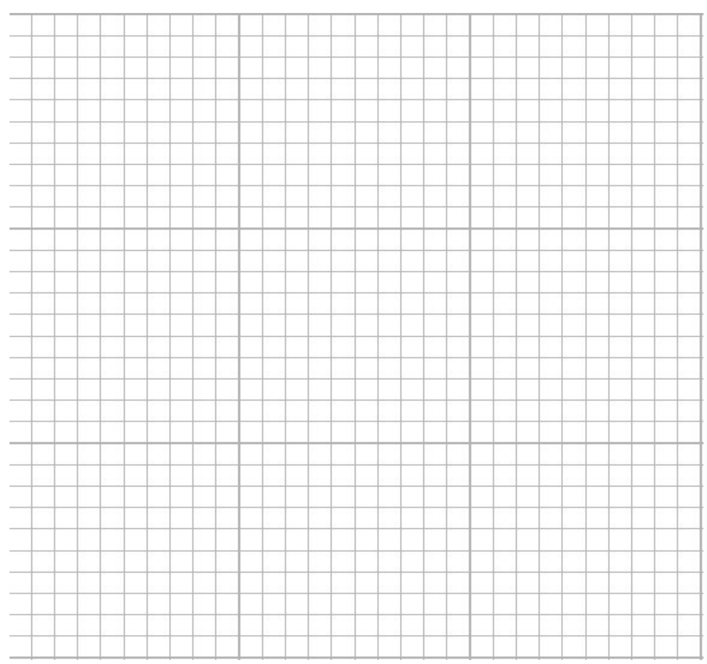
$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a,b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

3. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function $f(x) = \cos x$, $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$



4. Find the average value of the function $f(x)=\frac{4(x^2+1)}{x^2}$, $[1,3]$ and all the values of x in the interval for which the function equals its average value.