MATH 250/GRACEY NAME______________________________
WORKSHEET/5.1

DEFINITION OF THE NATURAL LOGARITHMIC FUNCTION

The **natural logarithmic function** is defined by

$$\ln x = \int_1^x \frac{1}{t}\,dt, \quad x > 0$$

The domain of the natural logarithmic function is the set of all **positive** real numbers, $(0.\infty)$.

THEOREM: LOGARITHMIC PROPERTIES

If a and b are positive numbers and n is rational, then the following properties are true:

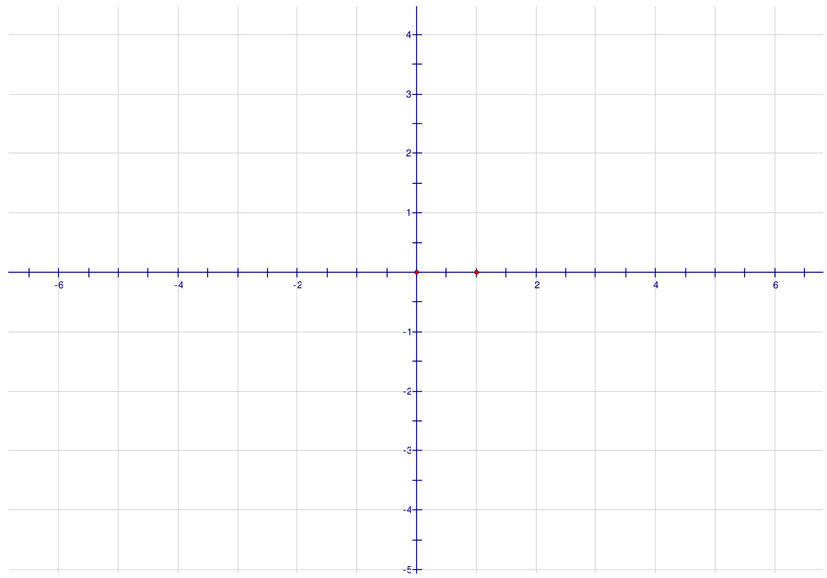
1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

1. Sketch the graph of the function and state its domain and range.

$$f(x) = \ln(x-1)$$

Domain:

Range:



2. Use the properties of logarithms to expand the logarithmic expression.

a. $\ln \frac{\sqrt[5]{x}}{y^2}$

b. $\ln\left(6e^3\right)$

3. Write the expression as a logarithm of a single quantity.

a. $\ln(x+4)+\ln(x-4)$

b. $\frac{1}{2}\left[3\ln x-\left(5\ln\left(x^3+2\right)+\ln x\right)\right]$

4. Find the limit.

a. $\lim_{x\to 6^-}\ln(6-x)$

b. $\lim_{x\to 5^+}\ln\frac{x}{\sqrt{x-4}}$

THEOREM: DERIVATIVE OF THE NATURAL LOGARITHMIC FUNCTION

Let u be a differentiable function of x.

1. $$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$$

2. $$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \quad u > 0$$

DERIVATIVE INVOLVING

ABSOLUTE VALUE

If u is a differentiable function of x

$$\frac{d}{dx}\left[\ln|u|\right] = \frac{u'}{u}$$

5. Find an equation of the tangent line to the graph of the logarithmic function $y = \ln x^{1/2}$ at the point $(1,0)$.

6. Find the derivative of the function.

a. $y = \ln\left(3x^4 - 5\right)$

b. $y = x \ln x$

c. $f(x) = \ln\left(\dfrac{6x}{6x-5}\right)$

d. $h(t) = \sqrt[4]{\dfrac{x-2}{x+2}}$

e. $y = \ln\sqrt{5 + \sin^2 x}$

f. $x^2 y - \ln(xy) = 8y$, find $\dfrac{dy}{dx}$.

7. Find the relative extrema and inflection points for the function $f(x) = \dfrac{\ln x}{x}$.

8. Use logarithmic differentiation to find dy/dx.

$$y = \sqrt{(x-1)(x-2)(x-3)}$$