MATH 250/GRACEY
WORKSHEET/5.4-5.5

NAME________________________________

$$\ln\left(e^x\right) = x \quad \text{and} \quad e^{\ln x} = x$$

Properties of Logarithmic Functions to Base a

1. $\log_a 1 = 0$
2. $\log_a xy = \log_a x + \log_a y$
3. $\log_a x^n = n \log_a x$
4. $\log_a \frac{x}{y} = \log_a x - \log_a y$

Properties of Inverse Functions

1. $y = a^x \Leftrightarrow x = \log_a y$
2. $a^{\log_a x} = x, \ x > 0$
3. $\log_a a^x = x, \ \forall x$

1. Evaluate without using a calculator.

a. $\log_6 36$

b. $\log_3 \frac{1}{81}$

2. Write the exponential equation as a logarithmic equation or vice versa.

a. $\log_2 256 = 8$

b. $49^{1/2} = 7$

3. Solve the equation. Round to the nearest thousandth.

a. $3\left(5^{x-1}\right) = 86$

b. $\log\left(\sqrt{x-5}\right) = 4.8$

c. $(\ln x)^2 - \ln x^3 = 0$

- DERIVATIVES OF EXPONENTIAL FUNCTIONS

Theorem: Derivative of the Natural Exponential Function

Let u be a differentiable function of x.

1. $$\frac{d}{dx}\left[e^x\right]=e^x$$

2. $$\frac{d}{dx}\left[e^u\right]=e^u\frac{du}{dx}$$

4. Find the derivative of each function.

a. $f(x)=e^{-\sqrt{x}}$

b. $y=e^{\ln x^2}$

c. $g(t)=\left(e^{-2t}-e^t\right)^3$

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d. $f(x) = xe^x$

e. $y = \dfrac{e^{-x} - e^x}{e^{-x} + e^x}$

- Derivatives for Bases Other than *e*

Theorem: Derivative for bases other than e

Let *a* be a positive real number and let u be a differentiable function of x.

1. $$\frac{d}{dx}\left[a^x\right] = (\ln a)\, a^x$$

2. $$\frac{d}{dx}\left[a^u\right] = (\ln a)\, a^u \frac{du}{dx}$$

3. $$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{(\ln a)\, x}$$

f. $y = \frac{3^{2x}}{x}$

g. $g(\theta) = 4^{\theta} \sin(\pi\theta)$

h. $y = \frac{\log_4 x^{10}}{e^{5x^2}}$

i. $r(s) = \sqrt[6]{s^5} \log_3 \sqrt{1-s}$

- INTEGRALS OF EXPONENTIAL FUNCTIONS

Theorem: Integration Rules for Exponential Functions

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$

2. $\int e^u du = e^u + C$

5. Integrate.

a. $\int \frac{2e^x - 2e^{-x}}{\left(e^x + e^{-x}\right)^2} dx$

b. $\int \frac{e^{x^{-2}}}{x^3} dx$

Definition of Logarithmic Function to Base a

If a is a positive real number $(a \neq 1)$ and x is any positive real number, then the **logarithmic function to the base** a is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{1}{\ln a} \ln x$$

Definition of Exponential Function the Base a

If a is a positive real number $(a \neq 1)$ and x is any real number, then the **exponential function to the base** a is denoted by a^x and is defined by

$$a^x = e^{(\ln a)x}$$

If $a = 1$, then $y = 1^x = 1$ is a constant function.

- Integrating
  - Option 1: Convert to base *e* using the formula $a^x = e^{(\ln a)x}$ and then integrate or
  - Option 2: Integrate directly using the integration formula

$$\int a^x dx = \left(\frac{1}{\ln a}\right) a^x + C$$

c. $\int 5^x dx$

d. $\int \frac{3^{2x}}{1+3^{2x}} dx$

e. $\int x^2 8^{-x^3} dx$

6. Find the area of the region bounded by the graphs of the equations $y = e^{-2x} + 2,\ y = 0,\ x = 0,\ \text{and } x = 2.$

7. Find an equation for the tangent line to the graph of $y = \log(2x)$ at the point $(5,1)$.