MATH 250/GRACEY NAME_____

APPLICATIONS REVIEW WORKSHEET

1. You drive to Las Vegas at a rate of 85 mph. On the return trip, you drive at a rate of 70 mph. What is your average speed for the entire trip?

2. For a quantity of gas at a constant temperature, the pressure *P* is inversely proportional to the volume *V*. Find the limit of *P* as *V* approaches zero from the right.

3. The displacement from equilibrium of an object in harmonic motion on the end of a spring is $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$, where *y* is measured in feet and *t* is the time in seconds. Determine the position and velocity of the object when $t = \frac{\pi}{8}$.

4. Find all points on the circle $x^2 + y^2 = 25$ where the slope is $\frac{3}{4}$.

5. The radius r of a sphere is increasing at a rate of 5 inches per minute.

   a. Find the rate of change of the volume when

      i. $r = 10$ inches

      ii. $r = 15$ inches

   b. Explain why the rate of change of the volume of the sphere is not constant even though the rate of change of the radius is constant.

6. A conical tank (with vertex down) is 20 feet across the top and 10 feet deep. If water is flowing into the tank at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when the water is 6 feet deep.

7. A balloon rises at a rate of 9 feet per second from a point on the ground 900 feet from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 900 feet above the ground.

8. Locate the absolute extrema of the function $g(x) = \sec x$ on the closed interval $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$.

9. Consider the function $f(x)=\frac{x}{x^2+1}$.

a. Find the first derivative.

b. Find the second derivative.

c. Find any relative extrema.

d. Find any points of inflection.

e. On what interval(s) is the function increasing?

f. On what interval(s) is the function decreasing?

g. Discuss the concavity of the function.

10. On a given day, the flow rate F (cars per hour) on a congested roadway is $F = \frac{v}{22 + 0.02v^2}$ where v is the speed of the traffic in miles per hour. What speed will maximize the flow rate on the road?

11. Determine the dimensions of a rectangular solid with a square base with maximum volume if its surface area is 400 square centimeters.

12. A rectangular page is to contain 36 square inches of print. The margins on each side are to be $1\frac{1}{2}$ inches. Find the dimensions of the page such that the least amount of paper is used.

13. The measurement of the edge of a cube is found to be 14 inches, with a possible error of 0.02 inch. Use differentials to approximate the maximum possible propagated error in computing
    a. The volume of the cube.

    b. The surface area of the cube.

14. The measurement of the circumference of a circle is found to be 56 inches, with a possible error of 1.2 inches.
    a. Approximate the percent error in computing the area of the circle.

    b. Estimate the maximum allowable percent error in measuring the circumference if the error in computing the area cannot exceed 3%.

15. The median waiting time (in minutes) for people waiting for service in a convenience store is given by the solution of the equation $\int_0^x 0.3e^{-0.3t}dt = \frac{1}{2}$. Solve the equation.

16. A lake is stocked with 500 fish, and their population increases according to the logistic curve $p(t) = \frac{10000}{1+19e^{-t/5}}$ where t is measured in months.

a. What is the limiting size of the fish population?

b. At what rate is the fish population changing at
   i. The end of 1 month?

   ii. The end of 10 months?

c. After how many months is the population increasing most rapidly?

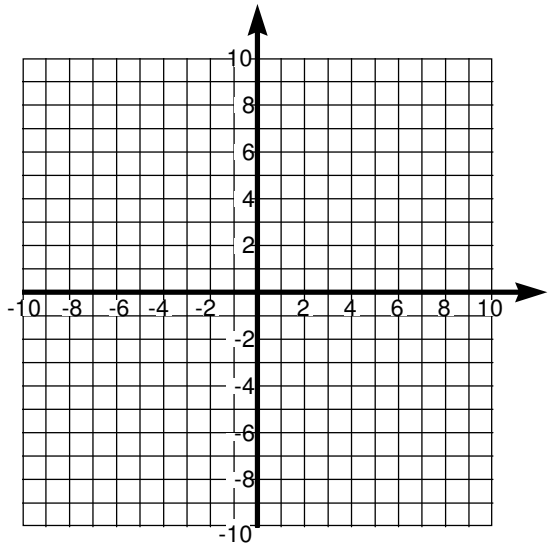
17. Compound Interest.

a. How large a deposit, at 7% interest compounded continuously, must be made in order to obtain a balance of $10000 in 15 years?

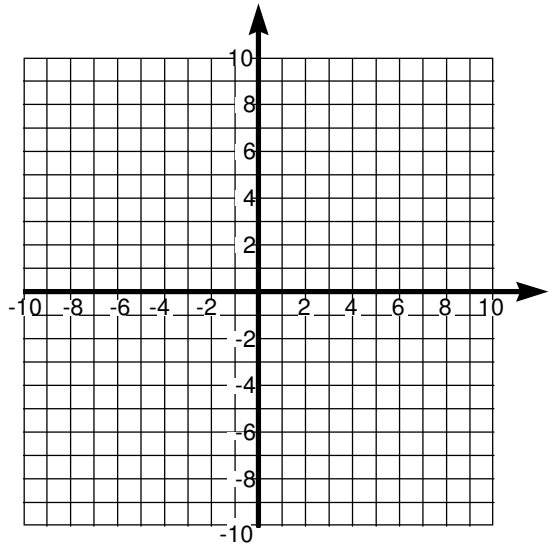
b. A deposit earns interest at a rate of *r* percent compounded continuously and doubles in value in 10 years. Find *r*.

18. Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

a. $f(x) = x^4 - 2x^2,\ g(x) = 2x^2$



b. $f(y) = y(2-y),\ g(y) = -y$



19. Set up and evaluate the definite integral that gives the area of the region bounded by the graph of the function and the tangent line to the graph at the given point.

$$y = x^3,\ (-1,1)$$