MATH 250/GRACEY
WORKSHEET/3.8-3.9

NAME________________________

NEWTON'S METHOD FOR APPROXIMATING THE ZEROS OF A FUNCTION

Let $f(c)=0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.

1. Make an initial estimate x_1 that is close to c. (Make a graph!)
2. Determine a new approximation $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$.
3. If $|x_n-x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to step 2 and calculate a new approximation.

Each successive application of this procedure is called an **iteration**.

1. Complete two iterations of Newton's Method for the functions using the initial guess.

$f'(x)=2x$

a. $f(x)=x^2-5,\ x_1=2.2$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$	$\lvert x_n-x_{n+1}\rvert$
1	2.2	−.16	4.4	$x_2=2.2-\frac{-.16}{4.4}=2.2364$	.0364
2	2.2364	.0015	4.4728	$x_3=2.2364-\frac{.0015}{4.4728}=2.2361$	.0003

$f'(x)=\sec^2 x$

b. $f(x)=\tan x,\ x_1=0.1$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$	$\lvert x_n-x_{n+1}\rvert$
1	0.1	.10033	1.0136	.00102	.09898
2	.00102	.00102	1	0	.00102

2. Find the point on the graph of $f(x)=x^2$ that is the closest to the point $(4,-3)$.

DEFINITION OF DIFFERENTIALS

Let $y=f(x)$ represent a function that is differentiable on an open interval containing *x*. The **differential of *x*** (denoted by *dx*) is any nonzero real number. The **differential of *y*** (denoted by *dy*) is $dy=f'(x)\,dx$. Another useful formula is $f(x+\Delta x)\approx f(x)+dy=f(x)+f'(x)\,dx$

3. Use the given information to evaluate and compare Δy and dy.

a. $y=1-2x^2,\ x=0,\ \text{and}\ \Delta x=dx=-0.1$

$1-2(-.1)^2=.98$
$1-2(0)^2=1$

$\Delta y=.98-1=0.02$
$dy=f'(x)dx$
$dy=0(-.1)=0$

$f'(x)=-4x$
$f'(0)=-4(0)=0$

b. $y=2x+1,\ x=2,\ \text{and}\ \Delta x=dx=0.01$

$y'=2$

$\Delta y=(2(2.01)+1)-(2(2)+1)$
$=5.02-5=.02$

$\Delta y=.02$
$dy=2(.01)=.02$

4. Find the differential *dy* of the given function.

a. $y = 3x^{2/3}$

$y' = \cancel{3}\left[\frac{2}{\cancel{3}}(x)^{-1/3}\right]$

$y' = \frac{2}{x^{1/3}}$

$$\boxed{dy \doteq \frac{2\,dx}{x^{1/3}}}$$

b. $y = x \sin x$

$y' = 1\sin x + x(\cos x)$

$y' = \sin x + x\cos x$

$$\boxed{dy \approx (\sin x + x\cos x)\,dx}$$

Estimation of errors propagated by physical measuring devices

$$f\Bigg(\underbrace{x + \overbrace{\Delta x}^{\substack{\text{measurement}\\\text{error}}}}_{\text{exact value}}\Bigg) - f\Bigg(\underbrace{x}_{\substack{\text{measured}\\\text{value}}}\Bigg) = \overbrace{\Delta y}^{\substack{\text{propagated}\\\text{error}}}$$

DIFFERENTIAL FORMULAS

Let u and v be differentiable functions of x.

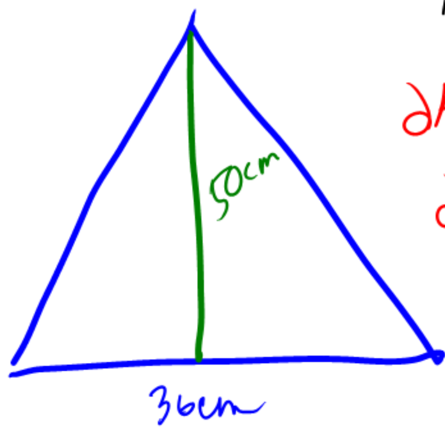
Constant Multiple: $d[cu] = c\,du$

Sum or Difference: $d[u \pm v] = du \pm dv$

Product: $d[uv] = u\,dv + v\,du$

Quotient: $d\left[\frac{u}{v}\right] = \frac{v\,du - u\,dv}{v^2}$

5. The measure of the base and the altitude of a triangle are found to be 36 and 50 centimeters, respectively. The possible error in each measurement is 0.25 centimeter. Use differentials to approximate the possible propagated error in computing the area of the triangle.



$$A = \frac{1}{2}bh$$

$$\partial A = \frac{1}{2}\left(b\,\partial h + h\,\partial b\right)$$

$$\partial A = \frac{1}{2}\left(36(.25) + 50(.25)\right)$$

$$\partial A = .5(21.5)$$

$$\boxed{\partial A = \pm 10.75}$$

6. The measurement of the edge of a cube is found to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible propagated error in computing…

a. The volume of the cube.

$$V = s^3$$

$$\partial V = 3s^2\,\partial s$$

$$\partial v = 3(12)^2(.03)$$

$$\partial v = 3(144)(.03)$$

$$\partial V = \pm 12.96\text{ in}^3$$

b. The surface area of the cube.

$$SA = 6s^2$$

$$\partial SA = 12s\,\partial s$$

$$\partial SA = 12(12)(.03)$$

$$\partial SA = 144(.03) = \pm 4.32\text{ in}^2$$