MATH 250/GRACEY

WORKSHEET/2.1-2.2

The limit definition of the derivative for some differentiable function f evaluated at x:

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Find the derivative of the following functions using the limit definition for the derivative.

1. $f(x)=-x^2+x-2$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-(x+\Delta x)^2+(x+\Delta x)-2-(-x^2+x-2)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{-x^2}-2x\Delta x-\Delta x^2+\cancel{x}+\Delta x\cancel{-2}\cancel{+x^2}\cancel{-x}\cancel{+2}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(-2x-\Delta x+1)}{\cancel{\Delta x}}=-2x-0+1=\boxed{-2x+1}$$

2. $s(t)=\dfrac{4}{t^2}$

$$s'(t)=\lim_{\Delta t\to 0}\left(\frac{\frac{4}{(t+\Delta t)^2}-\frac{4}{t^2}}{\Delta t}\right)=\lim_{\Delta t\to 0}\left(\frac{4t^2-4(t+\Delta t)^2}{\Delta t\, t^2(t+\Delta t)^2}\right)$$

$$=\lim_{\Delta t\to 0}\frac{\cancel{4t^2}\cancel{-4t^2}-8t\cancel{\Delta t}-4\Delta t^{\cancel{2}}}{\cancel{\Delta t}\, t^2(t+\Delta t)^2}=\lim_{\Delta t\to 0}\frac{-8t-4\Delta t}{t^2(t+\Delta t)^2}=-\frac{8\cancel{t}}{t^{\cancel{4}3}}=\boxed{\frac{-8}{t^3}}$$

3. $y=-1$

$$y'=\lim_{\Delta x\to 0}\frac{(-1)-(-1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{0}{\Delta x}=\lim_{\Delta x\to 0}0=\boxed{0}$$

Use the following "short-cut" rules to evaluate the derivative of the following functions. Fully simplify your result, writing as a single rational expression with positive exponents, when applicable. Identify which rule(s) you use to find each derivative.

Constant rule: $\frac{d}{dx}[c]=0$

Power rule: $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$

Constant multiple rule: $\frac{d}{dx}\left[cf(x)\right]=cf'(x)$

The sum/difference of functions $\frac{d}{dx}\left[f(x)\pm g(x)\right]=f'(x)\pm g'(x)$

The sine and cosine functions: $\frac{d}{dx}[\sin x]=\cos x$ $\qquad \frac{d}{dx}[\cos x]=-\sin x$

4. $y=3x^2$

$\boxed{y'=6x}$

5. $f(x)=\sqrt{x}-2$

$f'(x)=\frac{1}{2}x^{-1/2}-0=\boxed{\frac{1}{2\sqrt{x}}}$

6. $g(t)=-\frac{1}{2t}+\frac{5}{t^3}=-\frac{1}{2}t^{-1}+5t^{-3}$

$g'(t)=\frac{1}{2}t^{-2}-15t^{-4}=\frac{1}{2}t^{-4}(t^2-30)=\boxed{\frac{t^2-30}{2t^4}}$

7. $y=3x^{2/3}-8x^{3/4}+10x^{4/5}$

$y'=2x^{-1/3}-6x^{-1/4}+8x^{-1/5}=2x^{-20/60}-6x^{-15/60}+8x^{-12/60}$

$=2x^{-20/60}\left[1-3x^{5/60}+4x^{8/60}\right]=\boxed{\frac{2\left(1-3x^{1/12}+4x^{2/15}\right)}{x^{1/3}}}$

8. Find the equation of the tangent line at $t=\frac{\pi}{3}$ for the function

$s(t)=5+\sin t-3\cos t$.

$s\left(\frac{\pi}{3}\right)=5+\frac{\sqrt{3}}{2}-3\left(\frac{1}{2}\right)$

$=\frac{7+\sqrt{3}}{2}$

$s'(t)=0+\cos t-3(-\sin t)$

$s'(t)=\cos t+3\sin t$

$s'\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)+3\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}+3\frac{\sqrt{3}}{2}=\frac{1+3\sqrt{3}}{2}$

$\boxed{y-\frac{7+\sqrt{3}}{2}=\frac{1+3\sqrt{3}}{2}\left(t-\frac{\pi}{3}\right)}$