MATH 250/GRACEY

WORKSHEET/2.3

Use the following "short-cut" rules to evaluate the derivative of the following functions. Identify $f(x)$ and $g(x)$. Fully simplify your result, writing as a single rational expression with positive exponents, when applicable.

Product rule: $\dfrac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$

Extension of the product rule:

$$\frac{d}{dx}\left[f(x)g(x)h(x)\right]=f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$$

Quotient rule: $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{f'(x)g(x)-f(x)g'(x)}{\left[g(x)\right]^2}$

Remaining trigonometric functions: $\dfrac{d}{dx}[\tan x]=\sec^2 x$ $\dfrac{d}{dx}[\cot x]=-\csc^2 x$

$\dfrac{d}{dx}[\sec x]=\sec x\tan x$ $\qquad$ $\dfrac{d}{dx}[\csc x]=-\csc x\cot x$

1. $y=x\cos x+\tan x$

$$y'=\left[1\cos x+x(-\sin x)\right]+\sec^2 x=\boxed{\cos x-x\sin x+\sec^2 x}$$

2. $h(x)=\dfrac{\sqrt{x}}{x^2-1}=\dfrac{x^{1/2}}{x^2-1}$

$$h'(x)=\frac{\frac{1}{2}x^{-1/2}(x^2-1)-x^{1/2}(2x)}{(x^2-1)^2}=\frac{\frac{1}{2}x^{3/2}-\frac{1}{2}x^{-1/2}-2x^{3/2}}{(x^2-1)^2}\cdot\frac{2}{2}$$

$$=\frac{-3x^{3/2}-x^{-1/2}}{2(x^2-1)^2}=-\frac{x^{-1/2}(3x^2-1)}{2(x^2-1)}=\frac{1-3x^2}{2x^{1/2}(x^2-1)}$$

3. $h(t)=\dfrac{\sqrt[4]{t}}{2\sqrt[4]{t}-1}=\dfrac{t^{1/4}}{2t^{1/4}-1}$

$$h'(t)=\frac{\frac{1}{4}t^{-3/4}(2t^{1/4}-1)-t^{1/4}(\frac{1}{2}t^{-3/4})}{(2t^{1/4}-1)^2}=\frac{\frac{1}{4}t^{-1/2}-\frac{1}{4}t^{-3/4}-\frac{2}{4}t^{-1/2}}{(2t^{1/4}-1)^2}$$

$$=\frac{-t^{-1/2}-t^{-3/4}}{4(2t^{1/4}-1)^2}=\frac{-t^{-3/4}(t^{1/4}+1)}{4(2t^{1/4}-1)^2}=\boxed{-\frac{t^{1/4}+1}{4t^{3/4}(2t^{1/4}-1)^2}}$$

4. $y=(x^2-1)\sec x\csc x$

$$y'=(2x)\sec x\csc x+(x^2-1)(\sec x\tan x)\csc x+(x^2-1)\sec x[-\csc x\cot x]$$

$$=\boxed{\sec x[2x\csc x+(x^2-1)\tan x\csc x-(x^2-1)\csc x\cot x]}$$

5. $y=(2x^3-5)^2=(2x^3-5)(2x^3-5)$

$$y'=(6x^2)(2x^3-5)+(2x^3-5)(6x^2)$$

$$\boxed{y'=2(2x^3-5)(6x^2)=12x^2(2x^3-5)}$$

6. Find the equation of the tangent line at $t=4$ for the function $s(t)=\dfrac{t}{1-t}$.

$$s'(t)=\frac{1(1-t)-t(-1)}{(1-t)^2}=\frac{1-t+t}{(1-t)^2}=\frac{1}{(1-t)^2}$$

$$s'(4)=\frac{1}{(1-(4))^2}=\frac{1}{(-3)^2}=\frac{1}{9}=\text{slope of tangent line}$$

$$s(4)=\frac{(4)}{1-(4)}=-\frac{4}{3}$$

$$\boxed{y+\frac{4}{3}=\frac{1}{9}(t-4)}$$