

Theorem: The Fundamental Theorem of Calculus

If a function  $f$  is continuous on the closed interval  $[a,b]$  and  $F$  is an antiderivative of  $f$  on the interval  $[a,b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

Guidelines for Using the Fundamental Theorem of Calculus

1. Provided you can find an antiderivative of  $f$ , you now have a way to evaluate a definite integral without having to use the limit of a sum.
2. When applying the Fundamental Theorem of Calculus, the following notation is convenient:

$$\begin{aligned} \int_a^b f(x) dx &= F(x) \Big|_a^b \\ &= F(b) - F(a) \end{aligned}$$

3. It is not necessary to include a constant of integration  $C$  in the antiderivative because

$$\begin{aligned} \int_a^b f(x) dx &= F(x) + C \Big|_a^b \\ &= [F(b) + C] - [F(a) + C] \\ &= F(b) - F(a) \end{aligned}$$

1. Evaluate the definite integral.

$$\begin{aligned} \text{a. } \int_{-2}^6 6dx &= 6x \Big|_{-2}^6 \\ &= 6[(6) - (-2)] \\ &= 6(8) \\ &= \boxed{48} \end{aligned}$$

$$\begin{aligned} \text{b. } \int_1^6 (2x^2 + 1)dx &= \left( \frac{2x^3}{3} + x \right) \Big|_1^6 \\ &= \left( \frac{2(6)^3}{3} + (6) \right) - \left( \frac{2(1)^3}{3} - (1) \right) \\ &= \frac{432}{3} + 6 - \frac{2}{3} + 1 = \frac{430}{3} + 7 = \boxed{\frac{451}{3}} \end{aligned}$$

$$\begin{aligned} \text{c. } \int_0^2 (2-t)\sqrt{t}dt &= \int_0^2 (2t^{1/2} - t^{3/2})dt = \left. \frac{2t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2} \right|_0^2 \\ &= \left( \frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2} \right) \Big|_0^2 = \left( \frac{4}{3}(2)^{3/2} - \frac{2}{5}(2)^{5/2} \right) - (0) \\ &= \frac{8}{3}\sqrt{2} - \frac{8}{5}\sqrt{2} = \frac{40\sqrt{2} - 24\sqrt{2}}{15} = \boxed{\frac{16\sqrt{2}}{15}} \end{aligned}$$

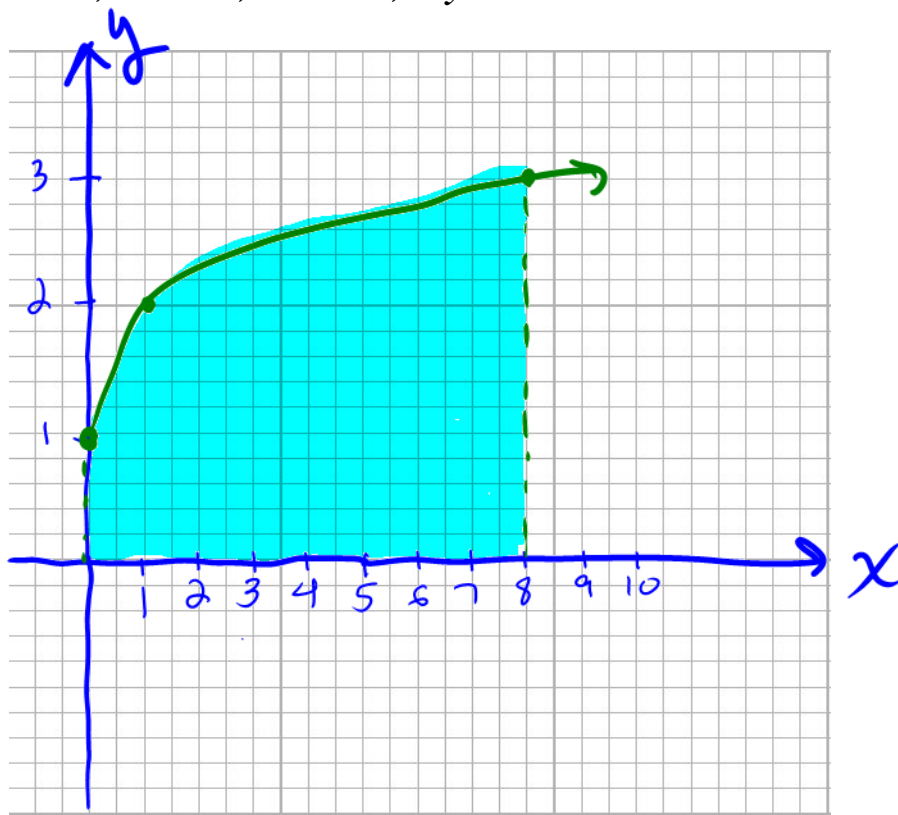
$$\text{d. } \int_1^4 (2v+5)^3 dv$$

$$(2v+5)^3 = 8v^3 + 60v^2 + 30v + 125$$

$$\begin{aligned} \int_1^4 (8v^3 + 60v^2 + 30v + 125)dv &= 2v^4 + 20v^3 + 15v^2 + 125v \Big|_1^4 \\ &= [2(256) + 20(64) + 15(16) + 125(4)] - [2 + 20 + 15 + 125] \\ &= 512 + 1280 + 240 + 500 - 162 = \boxed{2370} \end{aligned}$$

2. Find the area of the region bounded by the graphs of the equations.

$$y = 1 + \sqrt[3]{x}, \quad x = 0, \quad x = 8, \quad y = 0.$$



$$\begin{aligned} A &= \int_0^8 (1 + x^{1/3}) dx = \left( x + \frac{3}{4} x^{4/3} \right) \Big|_0^8 = \left( 8 + \frac{3}{4} (8)^{4/3} \right) - 0 \\ &= 8 + \frac{3}{4} \left( \frac{16}{1} \right) = \boxed{11} \end{aligned}$$

## THE MEAN VALUE THEOREM FOR INTEGRALS

If  $f$  is continuous on the closed interval  $[a, b]$ , then there exists a number  $c$  in the closed interval  $[a, b]$  such that

$$\int_a^b f(x) dx = f(c)(b-a)$$

### Definition of the Average Value of a Function on an Interval

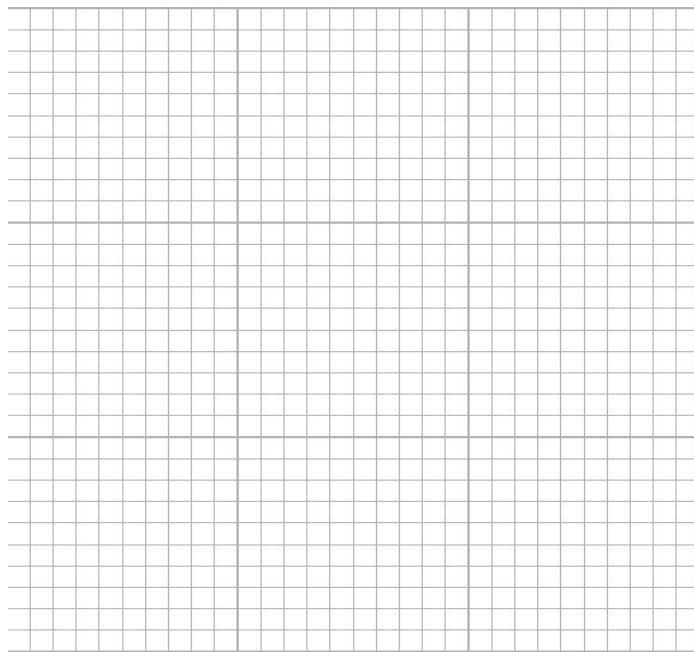
If  $f$  is integrable on the closed interval  $[a, b]$ , then the **average value** of  $f$  on the interval is

$$\frac{1}{b-a} \int_a^b f(x) dx$$

3. Find the value(s) of  $c$  guaranteed by the Mean Value Theorem for

Integrals for the function  $f(x) = \cos x$ ,  $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

See section 1  
notes from  
10/27



4. Find the average value of the function  $f(x) = \frac{4(x^2 + 1)}{x^2}$ ,  $[1, 3]$  and all the values of  $x$  in the interval for which the function equals its average value.

$$b-a = 3-1=2$$

$$\frac{1}{2} \int_1^3 \frac{4(x^2+1)}{x^2} dx = \frac{4}{2} \int_1^3 (1+x^{-2}) dx$$

$$= 2 \left[ x + \left( \frac{x^{-1}}{-1} \right) \right]_1^3$$

$$= 2 \left[ \left( 3 - \frac{1}{3} \right) - \left( 1 - \frac{1}{1} \right) \right]$$

$$= 2 \left[ \frac{8}{3} - 0 \right]$$

$$= \boxed{\frac{16}{3}}$$

Average  
value: