MATH 250/GRACEY NAME Key

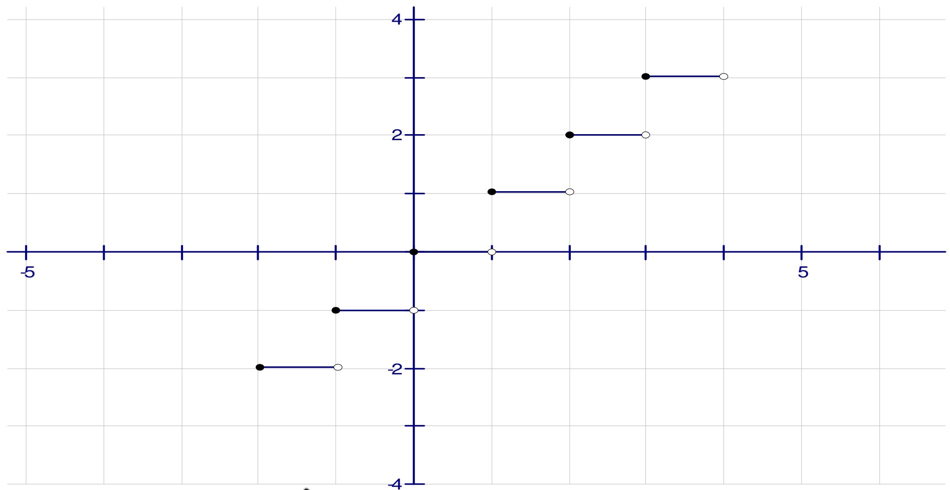
EXAM 1/CHAPTER 1

100 POINTS POSSIBLE DATE ____________

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!

LEAVE YOUR ANSWERS EXACT (NO DECIMALS)!

1. Use the graph of $f(x)$ shown below to find each limit, if it exists. If the limit does not exist, explain why.



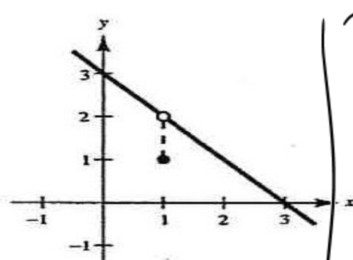
a. $\lim_{x\to -0.1} f(x) = \boxed{-1}$

c. $\lim_{x\to 0^+} f(x) = \boxed{0}$

b. $\lim_{x\to 0^-} f(x) = \boxed{-1}$

d. $\lim_{x\to 0} f(x)$ DNE since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

2. Consider the function shown below. Is this function continuous at $x=1$? EXPLAIN using the definition of continuity at a point!



Conditions

① $f(1) = 1$ (exists)

② $\lim_{x\to 1} f(x) = 2$ (exists)

③ $f(1) \stackrel{?}{=} \lim_{x\to 1} f(x)$ NO! so not continuous at $x=1$

a. $f(2) = \boxed{1}$

b. $f(1) = \boxed{1}$

c. $\lim_{x\to 1} f(x) = \boxed{2}$

3. Find the limit L. Then find $\partial > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \partial$.

$c = 2$
$L = 8$
$f(x) = 3x+2$
$\varepsilon = 0.01$

$$\lim_{x\to 2}(3x+2) = 8$$

$0 < |x-2| < \delta$, $|(3x+2) - 8| < 0.01$

$|3x-6| < 0.01$

$3|x-2| < 0.01$

$|x-2| < \frac{0.01}{3} = \frac{1}{300} = \delta$

$\boxed{\delta = \frac{1}{300}}$

4. Find the **FINITE LIMIT**. If there is no finite limit, write DNE (does not exist).

a. $\lim_{x\to -27} \sqrt[3]{x} = \sqrt[3]{-27} = \boxed{-3}$

b. $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x-1)}{\cancel{(x-4)}(x+2)}$

$= \lim_{x\to 4} \frac{x-1}{x+2} = \frac{4-1}{4+2} = \boxed{\frac{1}{2}}$

c. $\lim_{x\to 0} \frac{\frac{1}{4}\sin\left(\frac{x}{4}\right)}{\frac{x}{4}} = \frac{1}{4}(1) = \boxed{\frac{1}{4}}$

d. $\lim_{x\to \pi/2} \tan x$ $\boxed{\text{DNE}}$

e. $\lim_{x\to -1} \frac{x+1}{x^3+1} = \lim_{x\to -1} \frac{\cancel{(x+1)}^{1}}{\cancel{(x+1)}(x^2-x+1)} = \lim_{x\to -1} \frac{1}{x^2-x+1}$

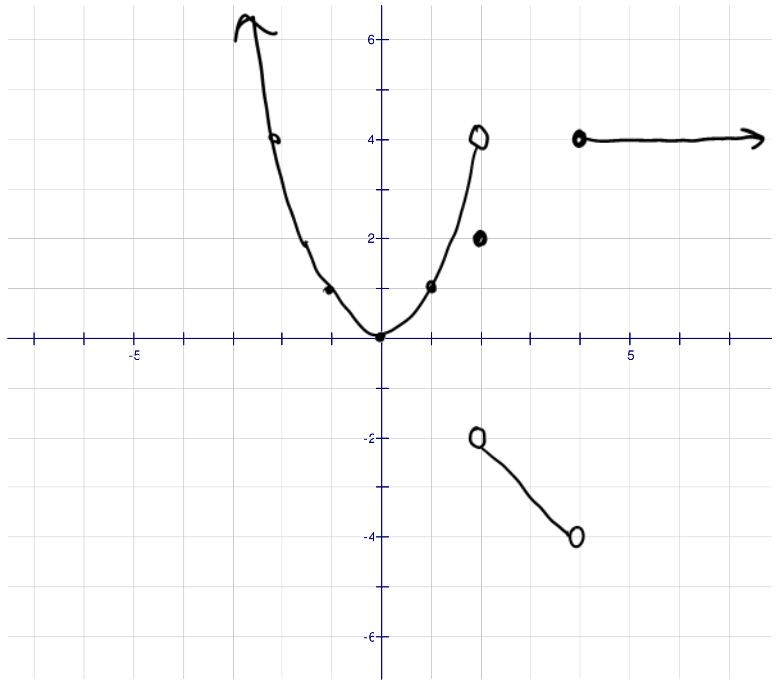
$= \frac{1}{(-1)^2-(-1)+1} = \frac{1}{1+1+1} = \boxed{\frac{1}{3}}$

f. $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} = \lim_{x\to 6} \frac{(x-2)-4}{(x-6)(\sqrt{x-2}+2)}$

$= \lim_{x\to 6} \frac{\cancel{x-6}^{1}}{\cancel{(x-6)}(\sqrt{x-2}+2)} = \frac{1}{\sqrt{6-2}+2} = \boxed{\frac{1}{4}}$

5. Sketch the following function.

$$f(x)=\begin{cases} x^2, \text{ if } x<2, \\ 2, \text{ if } x=2, \\ -x, \text{ if } 2<x<4 \\ 4, \text{ if } x\geq 4 \end{cases}$$



a) Identify the values of c, for which $\lim_{x\to c} f(x)$ exists. Use interval notation.

$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

b) On what interval(s) is this function continuous? Use interval notation.

$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$