MATH 250 PREPARATION

Simplify the following expressions and rewrite each expression as a single rational expression with positive exponents.

1. $2x\left(1-x^2\right)^{\frac{1}{2}}+x^2\left(1-x^2\right)^{-\frac{1}{2}}(-2x)$

$$= 2x\left(1-x^2\right)^{1/2} - 2x^3\left(1-x^2\right)^{-1/2}$$

$$= 2x\left(1-x^2\right)^{-1/2}\left[\left(1-x^2\right) - x^2\right]$$

$$= 2x\left(1-x^2\right)^{-1/2}\left(1-2x^2\right)$$

$$= \boxed{\frac{2x\left(1-2x^2\right)}{\left(1-x^2\right)^{1/2}}}$$

2. $\dfrac{6t^5\left(t^3+5\right)^{\frac{2}{5}}-t^6\left(t^3+5\right)^{-\frac{3}{5}}\left(3t^2\right)}{\left[\left(t^3+5\right)^{\frac{2}{5}}\right]^2}$

$$= \frac{6t^5\left(t^3+5\right)^{2/5} - 3t\left(t^3+5\right)^{-3/5}}{\left(t^3+5\right)^{4/5}}$$

$$= \frac{3t^5\left(t^3+5\right)^{-3/5}\left(2\left(t^3+5\right)-t^3\right)}{\left(t^3+5\right)^{4/5}}$$

$$= \boxed{\frac{3t^5\left(t^3+10\right)}{\left(t^3+5\right)^{7/5}}}$$

3. $\dfrac{\left(4x^2+5\right)(6x-1)-\left(3x^2-x+2\right)(8x)}{\left(4x^2+5\right)^2}$

$$= \frac{\cancel{24x^3} - 4x^2 + 30x - 5 - \cancel{24x^3} + 8x^2 - 16x}{(4x+5)^2}$$

$$= \boxed{\frac{4x^2+14x-5}{(4x+5)^2}}$$

4. $\dfrac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}$

$$= \frac{\cos x + \overbrace{\cos^2 x + \sin^2 x}^{1}}{(1+\cos x)^2}$$

$$= \frac{\overset{1}{\cancel{\cos x + 1}}}{(1+\cos x)^2}$$

$$= \boxed{\frac{1}{1+\cos x}}$$

5. $(3x+1)^6\left(\frac{1}{2\sqrt{2x-5}}\right)+6(3x+1)^5(3)\sqrt{2x-5}\left(\frac{2\sqrt{2x-5}}{2\sqrt{2x-5}}\right)$

$72x-180$

$=\frac{(3x+1)^6+36(3x+1)^5(2x-5)}{2\sqrt{2x-5}}=\frac{(3x+1)^5[(3x+1)+36(2x-5)]}{2\sqrt{2x-5}}$

$=\boxed{\frac{(3x+1)^5(75x-179)}{2\sqrt{2x-5}}}$

6. $(3z-1)^4\left[3(2z+5)^2(2)\right]+(2z+5)^3\left[4(3z-1)^3(3)\right]=6(3z-1)^4(2z+5)^2+12(2z+5)^3(3z-1)^3$

$=6(3z-1)^3(2z+5)^2[(3z-1)+2(2z+5)]$

$4z+10$

$=\boxed{6(3z-1)^3(2z+5)^2(7z+9)}$

Find $f(x)$ and $g(x)$ such that $h(x)=(f\circ g)(x)=f(g(x))$

7. $h(x)=\sqrt{5x^3-2}$

$f(x)=$ $\sqrt{x}$ $g(x)=$ $5x^3-2$

8. $h(x)=\sin^2 x$

$f(x)=$ $x^2$ $g(x)=$ $\sin x$

9. $h(x)=\frac{1}{6-x^2}$

$f(x)=$ $\frac{1}{x}$ $g(x)=$ $6-x^2$

10. $h(x)=(x^2-3x+4)^{3/2}$

$f(x)=$ $x^{3/2}$ $g(x)=$ $x^2-3x+4$