10/30/09

- lecture 9.5
- Practice problems from 9.4, 9.5

Monday

9.6
→ ratio & root tests
[great for factorials and nth roots]

Determine whether the infinite series converges or diverges

① $\sum_{n=1}^{\infty} \frac{2}{3^n-5} = \sum_{n=1}^{\infty} 2\left(\frac{1}{3^n-5}\right)$

Step 1: Nth term test for divergence

$\lim_{n\to\infty} \frac{2}{3^n-5} = 0$ More work!

Step 2: Strategize

Similar to $\sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{3}\right)^n$

LCT: $a_n = \frac{2}{3^n-5}$, $b_n = 2\left(\frac{1}{3}\right)^n$

conditions: a_n, b_n are greater than zero starting at $n=2$

test: $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{\cancel{2}}{3^n-5}\right)}{\left(\frac{\cancel{2}}{3^n}\right)}$

$$= \lim_{n\to\infty} \frac{3^n/3^n}{\frac{3^n}{3^n} - \frac{5}{3^n}}$$

$$= \lim_{n\to\infty} \frac{1}{1-\frac{5}{3^n}}$$

$$= \frac{1}{1-0}$$

$$= 1$$

Step 3: Conclusion

Since $\sum_{n=2}^{\infty} \frac{2}{3^n}$ is a convergent geometric series $[|r| = \frac{1}{3} < 1]$, the $\sum_{n=2}^{\infty} \frac{2}{3^n-5}$ and $\sum_{n=2}^{\infty} \frac{2}{3^n}$ have positive terms, and $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ [finite and positive], both series converge by the LCT.

2 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

Step 1: Identify a_n and then run the nth term test for divergence

$a_n = \frac{1}{n^{3/2}}$, $\lim_{n\to\infty} \frac{1}{n^{3/2}} = 0$

more work

passes cond. 1 of the AST

Step 2: Run the rest of the AST

① $\lim_{n\to\infty} \frac{1}{n^{3/2}} = 0$ ✓ ② $a_{n+1} = \frac{1}{(n+1)^{3/2}} < \frac{1}{n^{3/2}} = a_n$

So $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ converges

Det. absolute or cond. conv.

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n\sqrt{n}} \right| = \frac{1}{1^{3/2}} + \frac{1}{2^{3/2}} + \cdots + \frac{1}{n^{3/2}}$$

which is a convergent p-series, $p = \frac{3}{2} > 1$.

So... since $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n\sqrt{n}} \right|$ converges,

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ is absolutely convergent