10/5/09

- Lecture 8.4
  → trig. sub.
- Group Quiz

Prep for Wednesday

- review partial fraction decomposition!

MATH 251/GRACEY
WORKSHEET/8.4

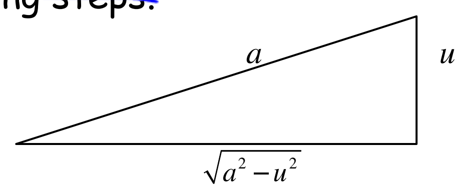
NAME

$\sin 2\theta = 2\sin\theta\cos\theta$

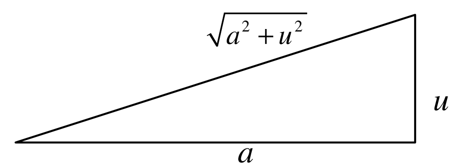
TRIGONOMETRIC SUBSTITUTION ($a > 0$)

~~Let $f(c)=0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.~~

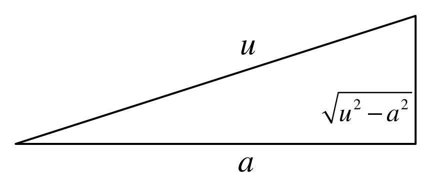
1. For integrals involving $\sqrt{a^2-u^2}$, let $u = a\sin\theta$.



2. For integrals involving $\sqrt{a^2+u^2}$, let $u = a\tan\theta$.



3. For integrals involving $\sqrt{u^2-a^2}$, let $u = a\sec\theta$.



Then $\sqrt{u^2-a^2} = \pm a\tan\theta$, where $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$. Use the positive value if $u > a$ and the negative value if $u < -a$.

1. Find the integral.

a. $\displaystyle\int \frac{t}{\left(1-t^2\right)^{3/2}}\,dt = \int \frac{t\,dt}{\left(\sqrt{1^2-t^2}\right)^3} = \int \frac{\sin\theta\,\cancel{\cos\theta}\,d\theta}{(\cos\theta)^2}$

$a = 1$
$u = t$

$t = 1\sin\theta \rightarrow \sqrt{1-t^2} = \sqrt{1-\sin^2\theta}$

$\sin\theta = \frac{t}{1}$

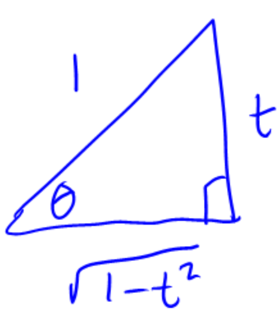
$= \sqrt{\cos^2\theta}$

$= \cos\theta$

$= \int \sec\theta\tan\theta\,d\theta$

$= \sec\theta + c$

$= \boxed{\frac{1}{\sqrt{1-t^2}} + c}$



$\frac{dt}{dt} = \frac{d\sin\theta}{dt}$

$1 = \frac{\cos\theta\,d\theta}{dt}$

$dt = \cos\theta\,d\theta$

b. $\int \frac{1}{\sqrt{x^2-9}}\,dx$

c. $\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$

$(2x)^2 = (3\tan\theta)^2$, $\frac{\partial}{\partial x} 2x = \frac{\partial}{\partial x} 3\tan\theta \rightarrow 2 = 3\sec^2\theta \frac{\partial\theta}{\partial x}$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$

$\sqrt{4x^2+9} = \sqrt{9\tan^2\theta + 9}$

$= \sqrt{9(1+\tan^2\theta)}$

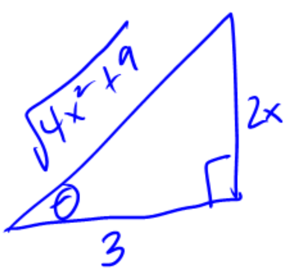
$= \sqrt{9\sec^2\theta}$

$= 3\sec\theta$

$2x = 3\tan\theta$

$x = \frac{3\tan\theta}{2}$

$x^4 = \frac{81}{16}\tan^4\theta$



$\tan\theta = \frac{2x}{3}$

So $$\int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{3\sec\theta}{\frac{81}{16}\tan^4\theta}\,\frac{3}{2}\sec^2\theta\,d\theta$$

$$= \frac{\cancel{9}}{\cancel{2}} \cdot \frac{\cancel{16}^{\,8}}{\cancel{81}_{\,9}} \int \frac{\sec^3\theta\,d\theta}{\tan^4\theta}$$

$$= \frac{8}{9} \int \frac{\frac{1}{\cancel{\cos^3\theta}}}{\frac{\sin^4\theta}{\cos^{\cancel{4}}\theta}}\,d\theta$$

$$\frac{\partial}{\partial\theta}\sin\theta = \cos\theta$$

$$= \frac{8}{9} \int \frac{\cos\theta}{(\sin\theta)^4}\,d\theta$$

$$= \frac{8}{9}\,\frac{(\sin\theta)^{-3}}{-3} + C$$

$$= -\frac{8}{27\sin^3\theta} + C$$

$$= -\frac{8}{27}\left(\frac{2x}{\sqrt{4x^2+3}}\right)^{-3} + C$$

$$= -\frac{\cancel{8}}{27}\left(\frac{(4x^2+3)^{3/2}}{\cancel{8}x^3}\right) + C$$

$$= -\frac{1}{27}\,\frac{(4x^2+3)^{3/2}}{x^3} + C$$

d. $\int e^{x}\sqrt{1-e^{2x}}\,dx$

e. $\int \frac{x^{3}+x+1}{x^{4}+2x^{2}+1}\,dx$