8/21/09

- Warm-up
- Finish 5.6

$u = f(x)$

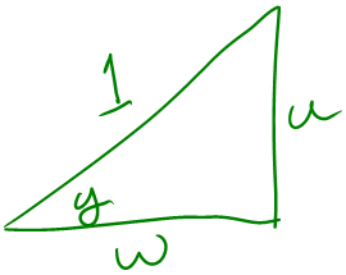
$y = \arcsin u \iff \sin y = u$

$\frac{\partial}{\partial x} \sin y = \frac{\partial}{\partial x} u$

$(\cos y)\frac{\partial y}{\partial x} = u'$

$\frac{\partial y}{\partial x} = \frac{u'}{\cos y}$ so $\frac{\partial y}{\partial x} = \frac{u'}{\left(\frac{\sqrt{1-u^2}}{1}\right)}$

$\sin y = \frac{u}{1}$



$w^2 + u^2 = 1^2$

$w = \sqrt{1-u^2}$

$\frac{\partial y}{\partial x} = \frac{u'}{\sqrt{1-u^2}}$

$\frac{\partial}{\partial x} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

$\frac{\partial}{\partial x}(\arcsin u) = \frac{u'}{\sqrt{1-u^2}}$

$\frac{\partial}{\partial x}(\arccos u) = -\frac{u'}{\sqrt{1-u^2}}$

$$\frac{\partial}{\partial x}(\arctan u) = \frac{u'}{1+u^2}$$

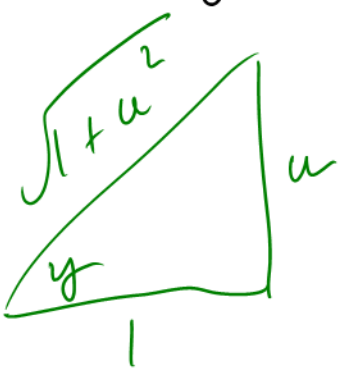
$$\frac{\partial}{\partial x}(\operatorname{arccot} u) = -\frac{u'}{1+u^2}$$

$$\frac{\partial}{\partial x}(\operatorname{arcsec} u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{\partial}{\partial x}(\operatorname{arccsc} u) = -\frac{u'}{|u|\sqrt{u^2-1}}$$

$\tan y = \overset{\tan}{(}\arctan u)$

$\tan y = u$



$$\frac{\partial}{\partial x}\tan y = \frac{\partial}{\partial x}u$$

$$(\sec^2 y)\frac{\partial y}{\partial x} = u'$$

$$\frac{\partial y}{\partial x} = \frac{u'}{(\sec y)^2}$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{u'}{\left(\frac{\sqrt{1+u^2}}{1}\right)^2} = \frac{u'}{1+u^2}$$

MATH 251/GRACEY NAME_____
WORKSHEET/5.6

1. Evaluate the expression without using a calculator.

a. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

b. $\arctan(1)$

c. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

d. $\operatorname{arccot}\left(\frac{2\sqrt{3}}{3}\right)$

e. $\tan\left(\arcsin\left(\frac{4}{5}\right)\right)$

f. $\sec(\arctan(x))$

2. Solve for x.

a. $\arccos(2x-\pi)=\frac{3}{4}$

b. $\arcsin x = \operatorname{arccsc} x$

3. Find the derivative of the function. **Write your result as a single ~~trigonometric~~ rational expression**.

a. $h(x)=3\arctan 2x$

$h'(x)=3\left(\frac{2}{1+(2x)^2}\right)$

$=\frac{6}{1+4x^2}$

b. $f(x)=x\arccos(x-1)$

$u = \frac{t}{4}$

$u' = \frac{1}{4}$

$u^2 = \frac{t^2}{16}$ 4

c. $g(t) = 16\arcsin\frac{t}{4} - t\sqrt{16-t^2}$

$$g'(t) = 16\left[\frac{\frac{1}{4}}{\sqrt{1-\frac{t^2}{16}}}\right] - \left[1\sqrt{16-t^2} + t\left(\frac{1}{2}(16-t^2)^{-1/2}\cdot(-2t)\right)\right]$$

$$= \frac{4}{\sqrt{\frac{16-t^2}{16}}} + \frac{t^2}{\sqrt{16-t^2}} - \sqrt{16-t^2}$$

$$= \frac{16 + t^2 - 16 + t^2}{\sqrt{16-t^2}} = \frac{2t^2}{\sqrt{16-t^2}}$$

d. $f(x) = \operatorname{arc\,sec}\sqrt{x}$

4. Consider the function $x^2 + x\arctan y = y - 1$ at the point $\left(-\frac{\pi}{4}, 1\right)$.

a. Find the slope of the line tangent to the function at the given point.

b. Find the equation of the line tangent to the function at the given point.