9/18/09

- 7.5
  ↳ Work

Monday

Have all questions relating to 5.6-5.8 and 7.1-7.5 ready

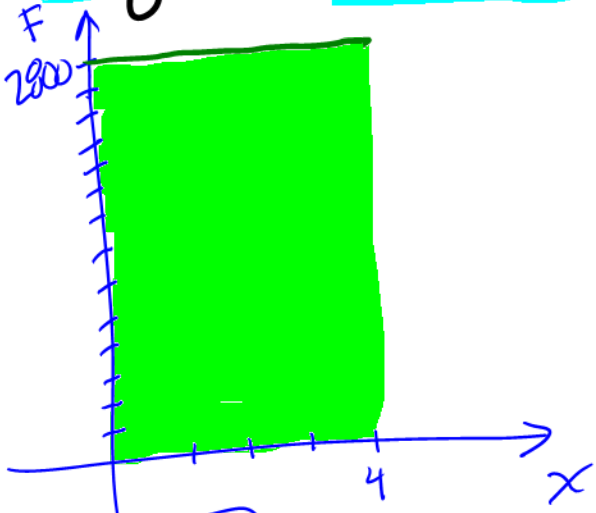
Wednesday

- EXAM 1/5.6-5.8 7.1-7.5
- HW due is 7.1-7.5

Constant Force

$$W = F \cdot D$$

7.5 #2 An electric hoist lifts a 2800-lb load 4 ft.



$$W = (2800 \text{ lbs})(4 \text{ ft})$$

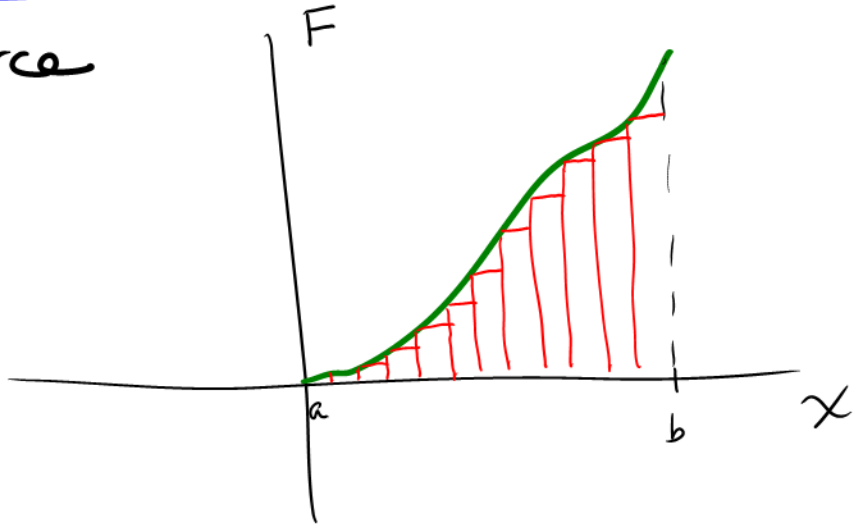
or

$$W = \int_0^4 2800 \, dx$$

$$W = 2800x \Big|_0^4 = 11{,}200 \text{ ft-pounds}$$

Variable Force

$$W = \int_a^b F(x) \, dx$$



Definition of Work Done by a Constant Force

If an object is moved a distance D in the direction of an applied constant force F then the **work** W done by the force is defined as $W = FD$.

4. Determine the work done in lifting a 100-pound bag of sugar 10 feet.

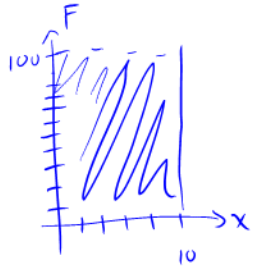
$W = Fd$

$W = \int_0^{10} 100\,dx$

$= 100x \Big|_0^{10}$

$= 100(10) - 100(0)$

$= 1000$ ft. lbs.



Definition of Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** W done by the force as the object is moved from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)\,dx.$$

Hook's Law: The force F required to compress or stretch a spring is proportional to the distance d that the spring is compressed or stretched from its original length.

$$F = kd$$

$F(x) = kx$

Newton's Law of Universal Gravitation: The force F of the attraction between two particles of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance d between the two particles.

for propulsion problems from earth to space

$C = m_1 m_2 \cdot k$

$$F = k\frac{m_1 m_2}{d^2}$$

$F(x) = \frac{C}{x^2}$

If m_1 and m_2 are given in grams and d is given in centimeters, F will be in dynes for a value of 6.670×10^{-8} cubic centimeter per gram-second squared.

Coulomb's Law: The force between two charges q_1 and q_2 in a vacuum is proportional to the product of the charges and inversely proportional to the square of the distance d between the two charges.

$$F = k\frac{q_1 q_2}{d^2}$$

If q_1 and q_2 are given in electrostatic units and d is given in centimeters, F will be in dynes for a value of $k = 1$.

Hookes law
$F(x) = kx$

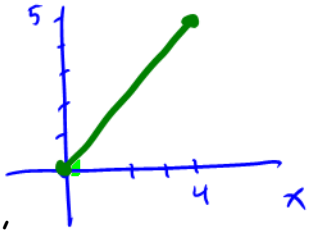
5. A force of 5 pounds compresses a 15-inch spring a total of 4 inches. How much work is done in compressing the spring 7 inches?

Step 1: Find k using given info to figure out our linear force function

$F(4) = 5 \rightarrow 5 = k \cdot 4$ so we have $k = \frac{5}{4}$ and $F(x) = \frac{5}{4}x$

Step 2: Evaluate the integral $W = \int_a^b F(x)\,dx$

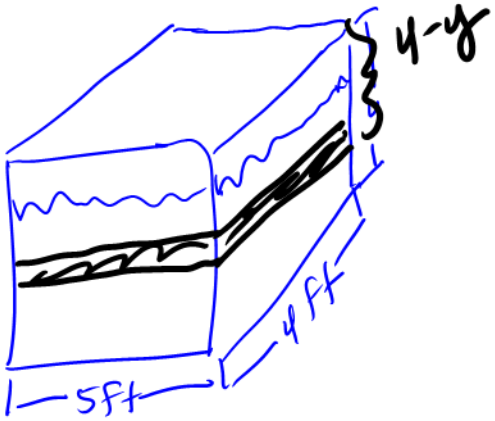
$W = \int \frac{5}{4}x\,dx$



6. Neglecting air resistance and the weight of the propellant, determine the work done in propelling a 5-ton satellite to a height of 100 miles above Earth. Use 4000 miles as the radius of the Earth.

$F(4000) = 5 \rightarrow 5 = \frac{C}{(4000)^2} \rightarrow C = 8 \times 10^7$

$W = \int_{4000}^{4100} 8 \times 10^7 / x^2\,dx$



$V = lwh$ ← y

$V = 5 \cdot 4 \cdot \Delta y$

$V = 20\,\Delta y$

weight of each layer

$62.4 \cdot V$

Draining entire tank

$$W = \int_0^4 62.4 \cdot 20(4-y)\,dy$$