10/30/09

- lecture 9.5
  ↳ Alternating Series
- Practice problems from 9.4, 9.5

Monday

9.6 → ratio and root tests

① $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n} - 1}$

Step 1: Nth term test for divergence

$\lim_{n\to\infty} \frac{1}{4\cdot\sqrt[3]{n} - 1} = 0 \rightarrow$ more work

Step 2: Strategize → DCT, $a_n \frac{1}{4\cdot\sqrt[3]{n} - 1}$, $b_n = \frac{1}{4\cdot\sqrt[3]{n}}$,

conditions: $a_n, b_n > 0$

$0 < b_n = \frac{1}{4n^{1/3}} < \frac{1}{4n^{1/3} - 1} = a_n$

$b_1 = \frac{1}{4} < \frac{1}{3} = a_n$

$b_8 = \frac{1}{8} < \frac{1}{7} = a_8$

test: Since $\frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ is a divergent p-series $[p = \frac{1}{3} \le 1]$ and $0 < b_n < a_n$,

$\frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^{1/3}-1}$ also diverges by the DCT.

② $\sum_{n=0}^{\infty}(-1)^n e^{-n}$

Step 1: Nth term test

$\lim_{n\to\infty}(-1)^n e^{-n} = 0$

more work

cond. 1 of the AST is satisfied

Step 2: Run AST → $a_n = e^{-n} = \frac{1}{e^n}$

① $\lim_{n\to\infty}\frac{1}{e^n} = 0$ ✓ / ② $0 < \frac{1}{e^{n+1}} \leq \frac{1}{e^n}$ ✓

Step 3: Conclusion

$\sum_{n=0}^{\infty}(-1)^n e^{-n}$ converges by the AST

Absolute or conditional convergence?

$$\left|(-1)^n e^{-n}\right| = e^{-n}$$

$\sum_{n=0}^{\infty}\frac{1}{e^n}$ is geometric with $|r| = \left|\frac{1}{e}\right| < 1$ which is convergent. So

Since $\sum_{n=0}^{\infty}\left|(-1)^n e^{-n}\right|$ converges, $\sum_{n=0}^{\infty}(-1)^n e^{-n}$ absolutely converges