10/5/09

- Lecture 8.4
  ↳ trig. sub.
- Group Quiz

Prep for wednesday

- review partial fraction decomposition

MATH 251/GRACEY NAME________________________

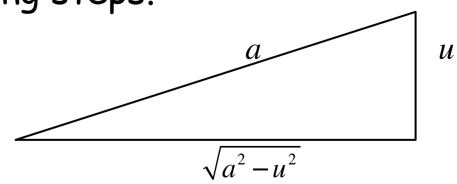
WORKSHEET/8.4

$\sin 2\theta = 2\sin\theta\cos\theta$

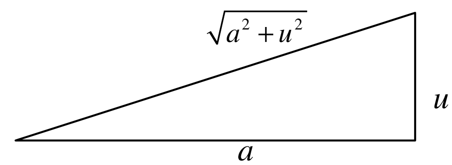
TRIGONOMETRIC SUBSTITUTION ($a>0$)

~~Let $f(c)=0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.~~

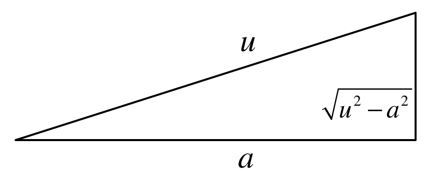
1. For integrals involving $\sqrt{a^2-u^2}$, let $u=a\sin\theta$.



2. For integrals involving $\sqrt{a^2+u^2}$, let $u=a\tan\theta$.



3. For integrals involving $\sqrt{u^2-a^2}$, let $u=a\sec\theta$.



Then $\sqrt{u^2-a^2}=\pm a\tan\theta$, where $0\le\theta<\frac{\pi}{2}$ or $\frac{\pi}{2}<\theta\le\pi$. Use the positive value if $u>a$ and the negative value if $u<-a$.

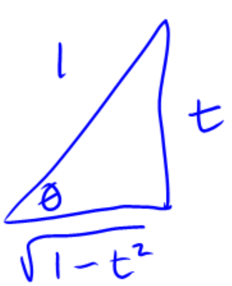
1. Find the integral.

a. $\int \frac{t}{(1-t^2)^{3/2}}\,dt \quad = \int \frac{t\,dt}{\left[\sqrt{1-t^2}\right]^3} = \int \frac{\sin\theta\,\cancel{\cos\theta}\,d\theta}{(\cos\theta)^{2}}$

$= \int \sec\theta\tan\theta\,d\theta$

$= \sec\theta + C$

$= \boxed{\frac{1}{\sqrt{1-t^2}} + C}$



$t = 1\sin\theta$

$\sin\theta = \frac{t}{1}$

$\sqrt{1-t^2} = \sqrt{1-\sin^2\theta}$

$= \sqrt{\cos^2\theta}$

$= \cos\theta$

$\frac{d}{dt}t = \frac{d}{dt}\sin\theta \rightarrow 1 = \cos\theta\frac{d\theta}{dt}$

$dt = \cos\theta\,d\theta$

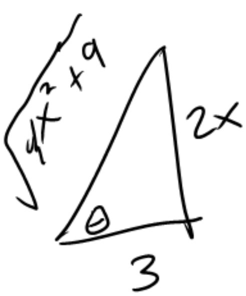
b. $\int \frac{1}{\sqrt{x^2-9}}\,dx$

c. $\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$

$2x = 3\tan\theta$ $\qquad \frac{dx}{dx} = \frac{d}{dx}\frac{3}{2}\tan\theta$

$x = \frac{3}{2}\tan\theta$ $\qquad 1 = \frac{3}{2}\sec^2\theta\,\frac{d\theta}{dx}$

$\tan\theta = \frac{2x}{3}$ $\qquad dx = \frac{3}{2}\sec^2\theta\,d\theta$



$$\sqrt{4x^2+9} = \sqrt{9\tan^2\theta + 9}$$

$$= \sqrt{9(\tan^2\theta + 1)}$$

$$= \sqrt{9\sec^2\theta}$$

$$= 3\sec\theta$$

$$\int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{(3\sec\theta)\left(\frac{3}{2}\sec^2\theta\,d\theta\right)}{\left[\frac{3}{2}\tan\theta\right]^4}$$

$$= \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{8}{\cancel{16}}}{\underset{9}{\cancel{81}}} \int \frac{\sec^3\theta \, d\theta}{\tan^4\theta}$$

$$= \frac{8}{9} \int \frac{\frac{1}{\cancel{\cos^3\theta}}}{\frac{\sin^4\theta}{\cos^{\cancel{4}}\theta}} \, d\theta$$

$$= \frac{8}{9} \int \frac{\cos\theta \, d\theta}{\sin^4\theta}$$

$$= \frac{8}{9} \left[\frac{\sin^{-3}\theta}{-3} \right] + C$$

$$= -\frac{8}{27\sin^3\theta} + C$$

$$= -\frac{8}{27} \left(\frac{1}{\left(\frac{2x}{\sqrt{4x^2+9}}\right)^3} \right) + C$$

$$= -\frac{\cancel{8}}{27} \frac{(4x^2+9)^{3/2}}{\cancel{8}x^3} + C$$

$$= -\frac{(4x^2+9)^{3/2}}{27x^3} + C$$

d. $\int e^x \sqrt{1-e^{2x}}\,dx$

e. $\int \frac{x^3+x+1}{x^4+2x^2+1}\,dx$