11/2/09

- Lecture 9.6
  - ↳ ratio and root tests

Prep for Wed.
- Read 9.7!
- Finish 9.4-9.6 hw

Friday
9.8 Power Series

Wed, 11/11 no school (veteran's day)
Fri 11/13, no class
* out of class assign. is a take-home exam covering 9.1-9.9 / due 11/16

9.6

(46) $\sum_{n=1}^{\infty} \left(\frac{n}{500}\right)^n$

(1) skip nth term test because the $\lim_{n\to\infty}$ is NASTY!

(2) root test

condition: $\sum_{n=1}^{\infty}\left(\frac{n}{500}\right)^n$ is a series. ✓

test: $\lim_{n\to\infty} \left|\left[\left(\frac{n}{500}\right)^n\right]\right|^{1/n} = \lim_{n\to\infty} \left|\frac{n}{500}\right| = \infty$

conclusion: $\sum_{n=1}^{\infty}\left(\frac{n}{500}\right)^n$ diverges by the root test.

(32) $\sum_{n=1}^{\infty} \frac{(-1)^n[2\cdot 4\cdot 6\cdots(2n)]}{2\cdot 5\cdot 8\cdots(3n-1)} = \sum_{n=1}^{\infty}\frac{(-1)^n(2n)}{(3n-1)}$

AST: $a_n = \frac{2n}{3n-1}$

conditions: $\lim_{n\to\infty}\frac{2n}{3n-1} = \frac{2}{3} \neq 0$

conclusion: $\sum_{n=1}^{\infty}\frac{(-1)^n 2n}{3n-1}$ diverges by the nth term test

(28) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$

(1) Skip nth term test → bad deriv.

(2) Ratio test (since there's factorials in there)

condition: $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$ is has nonzero terms

test: $a_n = \frac{(n!)^2}{(3n)!}$, $a_{n+1} = \frac{[(n+1)!]^2}{[3(n+1)]!} = \frac{[(n+1)!]^2}{(3n+3)!}$

$$\frac{a_{n+1}}{a_n} = \left(\frac{(n+1)!\,(n+1)!}{(3n+3)!}\right) \div \left(\frac{(n!)(n!)}{(3n)!}\right)$$

$$= \frac{(n+1)\,n!\,(n+1)\,n!}{(3n+3)(3n+2)(3n+1)(3n)!} \cdot \frac{(3n)!}{n!\,n!}$$

$$= \frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)}$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)} \right| = 0 < 1$$

conclusion: $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$ converges absolutely by the ratio test