8/24/09

- warm-up
- Start 5.7

Prep for Wed.

- Start 5.7 HW
- practice completing the square

Show that $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$

Think about $\int x\,dx = \frac{x^2}{2} + C$

if $y = \frac{x^2}{2} + C$, then

$$\frac{\partial}{\partial x} y = \frac{\partial}{\partial x}\left(\frac{x^2}{2} + C\right)$$

$$\frac{\partial y}{\partial x} = x + 0 \longrightarrow \int \partial y = \int x\,\partial x$$

$$y = \int x\,dx$$

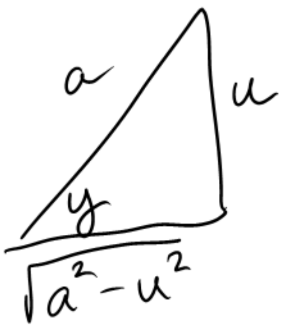
This is the process we'll be using for

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

Let $y = \arcsin\left(\frac{u}{a}\right)$

then $\sin y = \sin\left[\arcsin\left(\frac{u}{a}\right)\right]$

$$\frac{\partial}{\partial x}(\sin y) = \frac{\partial}{\partial x}\left(\frac{u}{a}\right) \longrightarrow \frac{\partial y}{\partial x}\cos y = \frac{\partial u}{\partial x}\cdot\frac{1}{a}$$



$$\frac{\partial y}{\partial x} = \frac{\partial u}{\partial x}\cdot\frac{1}{a}\cdot\frac{1}{\cos y}$$

$$\cancel{\partial x}\left(\frac{\partial y}{\cancel{\partial x}}\right) = \left(\frac{\partial u}{\cancel{\partial x}}\cdot\frac{1}{\cancel{a}}\cdot\frac{1}{\frac{\sqrt{a^2-u^2}}{\cancel{a}}}\right)\cancel{\partial x}$$

$$\int \partial y = \int \frac{\partial u}{\sqrt{a^2-u^2}}$$

$$\boxed{y = \int\frac{\partial u}{\sqrt{a^2-u^2}} + C}$$

MATH 251/GRACEY NAME________________________________
WORKSHEET/5.7

Derivatives of the inverse trigonometric functions. Let $u = f(x)$.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arccsc} u] = -\frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccot} u] = -\frac{u'}{1+u^2}$$

Integrals of the inverse trigonometric functions. Let $u = f(x)$.

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

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1. Find the integral.

a. $\int \frac{2}{\sqrt{3-x^2}}dx = 2\int \frac{dx}{\sqrt{(\sqrt{3})^2-(x)^2}}$

$a = \sqrt{3}$
$u = x$
$du = dx$

$= 2\int \frac{du}{\sqrt{(\sqrt{3})^2-(u)^2}}$

$= 2\arcsin\left(\frac{u}{\sqrt{3}}\right) + c$

$= \boxed{2\arcsin\left(\frac{x}{\sqrt{3}}\right) + c}$

b. $\int \frac{e^x}{16+e^{2x}}dx = \int \frac{\cancel{e^x}}{4^2+u^2}\left(\frac{du}{\cancel{e^x}}\right)$

$a^2 = 16$
$a = 4$
$u^2 = (e^x)^2$
$u = e^x$
$du/dx = e^x \rightarrow dx = \frac{du}{e^x}$

$= \int \frac{du}{4^2+u^2}$

$= \frac{1}{4}\arctan\frac{u}{4} + C$

$= \boxed{\frac{1}{4}\arctan\left(\frac{e^x}{4}\right) + C}$

$u = \sqrt{x}$

$u^2 = x$

$a^2 = 4$

$a = 2$

c. $\int \frac{5}{3x\sqrt{x-4}}dx$ $= \frac{5}{3}\int \frac{2\sqrt{x}\,du}{u^2\sqrt{u^2-2^2}} = \frac{10}{3}\int \frac{\cancel{u}\,du}{u^{\cancel{2}}\sqrt{u^2-2^2}}$

$= \frac{10}{3}\int \frac{du}{u\sqrt{u^2-2^2}} = \frac{\cancel{10}\,5}{3}\left[\frac{1}{\cancel{2}}\operatorname{arcsec}\frac{|u|}{2}\right] + C$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \rightarrow dx = 2\sqrt{x}\,du$

$= \boxed{\frac{5}{3}\operatorname{arcsec}\left(\frac{|\sqrt{x}|}{2}\right) + C}$

d. $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}$

e. $\int \frac{2x-5}{x^2+2x+2}dx$

f. $\int \frac{x+2}{\sqrt{-x^2-4x}}dx$