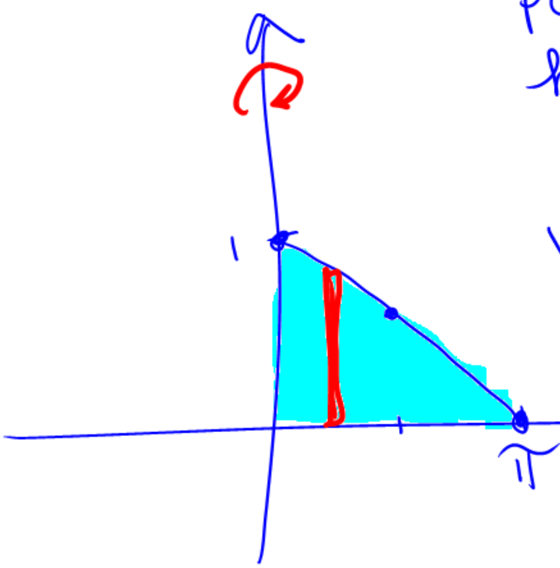
9/14/09	Wednesday	9/23/09
• HW ?'s • Start 7.4	• Finish 7.4 • Do all HW problems relating arc length	• Exam 1 / 5.6–5.8, 7.1–7.5

(12) 7.3

$$y = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}, \quad x = 0,\ y = 0,\ x = \pi$$



$p(x) = x - 0 = x$

$h(x) = \frac{\sin x}{x} - 0 = \frac{\sin x}{x}$

$V = 2\pi \int_0^{\pi} x \left(\frac{\sin x}{x}\right) dx$

$V = 2\pi \int_0^{\pi} \sin x \, dx$

$V = -2\pi \left(\cos x \Big|_0^{\pi} \right.$

$V = -2\pi \left((-1) - (1) \right)$

$V = -2\pi(-2)$

$\boxed{V = 4\pi}$

$$y = \frac{x^5}{10} + \frac{1}{6x^3}$$

Find y', then $1 + (y')^2$, then $\sqrt{1 + (y')^2}$

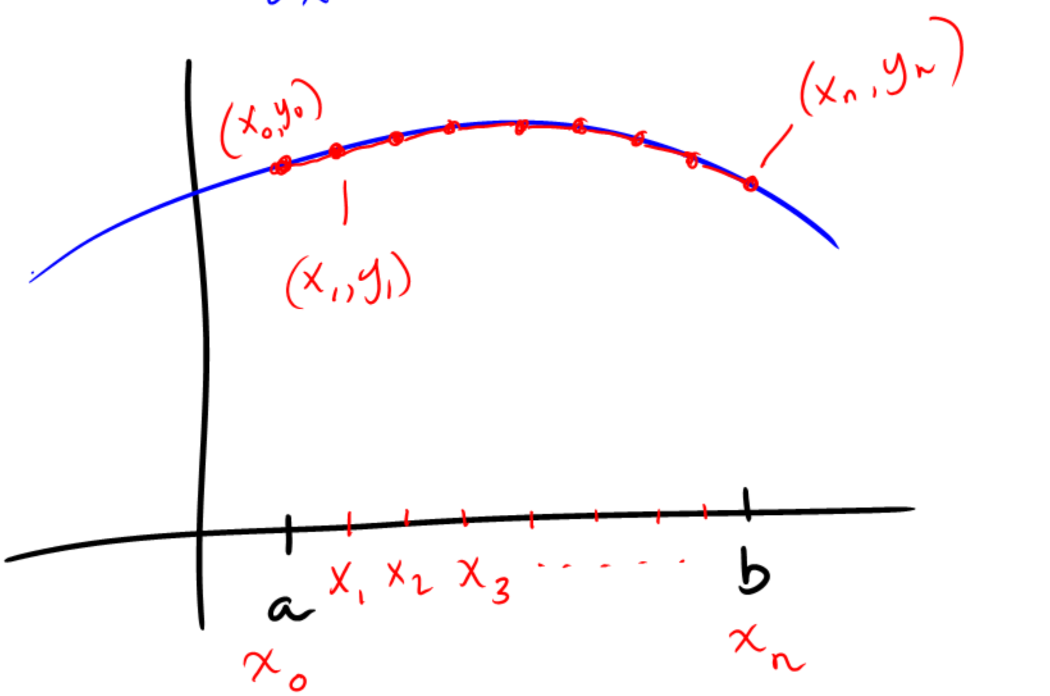
$$y' = \frac{5x^4}{10} + \frac{-3}{6x^4} = \frac{x^4}{2} - \frac{1}{2x^4}$$

$$(y')^2 = \left(\frac{x^4}{2}\right)^2 - 2\left(\frac{x^4}{2}\right)\left(\frac{1}{2x^4}\right) + \left(\frac{1}{2x^4}\right)^2$$

$$(y')^2 = \frac{x^8}{4} - \frac{1}{2} + \frac{1}{4x^8}$$

$$1 + (y')^2 = \frac{x^8}{4} + \frac{1}{2} + \frac{1}{4x^8} = \left(\frac{x^4}{2} + \frac{1}{2x^4}\right)^2$$

$$\sqrt{1 + (y')^2} = \frac{x^4}{2} + \frac{1}{2x^4}$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$d = \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}(\Delta x_i)^2}$$

$$d = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2\left[1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2\right]}$$

$$d = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\ \Delta x_i$$

arc length = $s = \int_a^b \sqrt{1+(y')^2}\ dx$

Now back to the warm-up...

$$y = \frac{x^5}{10} + \frac{1}{6x^3}, \quad [1, 2]$$

Find the arc length.

$$s = \frac{1}{2}\int_1^2 (x^4 + x^{-4})\,dx$$

$$s = \frac{1}{2}\left(\frac{x^5}{5} - \frac{1}{3x^3}\right)\Bigg|_1^2$$

$s = \frac{779}{240}$ linear units

MATH 251/GRACEY NAME____________________________________
WORKSHEET/CHAPTER 7.4-7.5

ARC LENGTH AND AREA OF A SURFACE OF REVOLUTION

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a,b]$. The arc length of f between a and b is

$$s = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx,\ y \text{ is a function of } x,$$

If $x = g(y)$ on the interval $[c,d]$, then the arc length of g between c and d is

$$s = \int_c^d \sqrt{1+\left[g'(y)\right]^2}\,dy,\ x \text{ is a function of } y$$

1. Find the arc length of the graph of the function $y = \frac{3}{2}x^{2/3}$ over the interval $[1,4]$.

2. Find the arc length of the graph of the function $y = \frac{x^3}{6} + \frac{1}{2x}$ over the interval $\left[\frac{1}{2}, 2\right]$.

Let $y = f(x)$ have a continuous derivative on the interval $[a,b]$. The area S of the surface of revolution formed by revolving the graph of about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1+\left[f'(x)\right]^2}\,dx,$$

y is a function of x, where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c,d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1+\left[g'(y)\right]^2}\,dy,$$

x is a function of y, where $r(y)$ is the distance between the graph of g and the axis of revolution.

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3. Find the **area of the surface** formed by revolving the graph of $f(x) = x^2$ on the interval $\left[0, \sqrt{2}\right]$ about the y-axis.

Definition of Work Done by a Constant Force

If an object is moved a distance *D* in the direction of an applied constant force *F* then the **work** *W* done by the force is defined as $W = FD$.

4. Determine the work done in lifting a 100-pound bag of sugar 10 feet.

Definition of Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** *W* done by the force as the object is moved from $x=a$ to $x=b$ is

$$W = \int_a^b F(x)dx.$$

Hook's Law: The force *F* required to compress or stretch a spring is proportional to the distance *d* that the spring is compressed or stretched from its original length.

$$F = kd$$

Newton's Law of Universal Gravitation: The force *F* of the attraction between two particles of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance *d* between the two particles.

$$F = k\frac{m_1 m_2}{d^2}$$

If m_1 and m_2 are given in grams and *d* is given in centimeters, *F* will be in dynes for a value of 6.670×10^{-8} cubic centimeter per gram-second squared.

Coulomb's Law: The force between two charges q_1 and q_2 in a vacuum is proportional to the product of the charges and inversely proportional to the square of the distance *d* between the two charges.

$$F = k\frac{q_1 q_2}{d^2}$$

If q_1 and q_2 are given in electrostatic units and *d* is given in centimeters, *F* will be in dynes for a value of $k = 1$.

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5. A force of 5 pounds compresses a 15-inch spring a total of 4 inches. How much work is done in compressing the spring 7 inches?

6. Neglecting air resistance and the weight of the propellant, determine the work done in propelling a 5-ton satellite to a height of 100 miles above Earth. Use 4000 miles as the radius of the Earth.