

TAKE-HOME EXAM IS DUE MONDAY NOVEMBER 16TH

TURN IN ONE EXAM PER GROUP/NO LATE EXAMS WILL BE ACCEPTED—NO EXCEPTIONS!!!

SHOW ALL WORK, GIVE EXACT ANSWERS AND SUPPORT ALL RESULTS TO EARN FULL

CREDIT

1. (6 points) A deposit of \$100 is made at the beginning of each month in an account at an annual interest rate of 3% compounded monthly. The balance in the account after n months is

$$A_n = 100(401)(1.0025^n - 1).$$

- a. Compute the first six terms of the sequence $\{A_n\}$.

- b. Is $\{A_n\}$ a convergent series? Explain.

- c. Find the balance in the account after 15 years.

2. (4 points) Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

a. $a_n = \frac{\sqrt{n}}{\ln n}$

b. $a_n = \frac{n^2}{e^n}$

3. (3 points) Write an expression for the n th term of the sequence.

$$1, \frac{5}{4}, 1, \frac{11}{16}, \frac{7}{16}, \frac{17}{64} \dots$$

4. (8 points) Find the sum of the convergent series.

a.
$$\sum_{n=3}^{\infty} \frac{3}{n^2 + n - 2}$$

b.
$$\sum_{n=1}^{\infty} \left[(0.2)^n + (-0.5)^n \right]$$

5. (5 points) You accept a job that pays a salary of \$90,000 for the first year. During the next 29 years you receive a 3% raise each year.
- What would your annual income be during the 23rd year?

- What would be your total compensation over the 30-year period?

6. (54 points) Determine the convergence or divergence of the series. Be sure to identify the test used and check that the conditions for using the test have been met.

- $$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$$

- $$\sum_{n=1}^{\infty} \frac{(n+2)!}{n!}$$

c. $\sum_{n=1}^{\infty} ne^{-n/2}$

d. $\sum_{n=1}^{\infty} (2 + 3\sqrt{n})^n$

e. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

f.
$$\sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$$

g.
$$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

h.
$$\sum_{n=3}^{\infty} \frac{(-1)^n \ln(n-2)}{n-2}$$

i.
$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+1}$$

7. (4 points) Find the 4th Taylor polynomial for $f(x) = (\ln x)(\cos x)$, centered at π .

8. (8 points) Consider the power series $\sum_{n=0}^{\infty} \frac{(-1)^n n! (x-4)^n}{3^n}$.

a. State where the power series is centered.

b. Find the radius of convergence.

c. Find the interval of convergence. Be sure to check for convergence at the endpoints of the interval.

9. (8 points) Use the power series $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ to determine a power series, centered

at 0, for the function $f(x) = \ln(x^2 + 1)$. Identify the interval of convergence.