

INDETERMINATE FORMS AND L'HÔPITAL'S RULE

Indeterminate Forms do not guarantee that a limit exists, nor do they indicate what the limit is, if one does exist. Earlier in your calculus life, you would rewrite the expression by using algebraic techniques. Unfortunately, not all indeterminate forms can be evaluated by algebraic manipulation.

The following forms have been identified as indeterminate:

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty, 0^0, 1^\infty, \infty^0$$

The following forms are determinate:

$$\infty + \infty \rightarrow \infty$$

$$-\infty - \infty \rightarrow -\infty$$

$$0^\infty \rightarrow 0$$

$$0^{-\infty} \rightarrow \infty$$

L'HÔPITAL'S RULE

Let f and g be functions that are differentiable on an open interval (a, b) containing c , except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b) , except possibly at c itself. If the limit of $\frac{f(x)}{g(x)}$ as x approaches c produces the indeterminate form $\frac{0}{0}$, then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

Provided the limit on the right exists (or is infinite). This result also applies if the limit of $\frac{f(x)}{g(x)}$ as x approaches c produces any one of the indeterminate forms ∞/∞ , $(-\infty)/\infty$, $\infty/(-\infty)$, or $(-\infty)/(-\infty)$.

Evaluate the following limits.

1.

$$\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1} .$$

2.

$$\lim_{x \rightarrow \infty} \frac{2x+1}{4x^2+x}$$

3.

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x - \pi}$$

4.

$$\lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 1}$$

5.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$