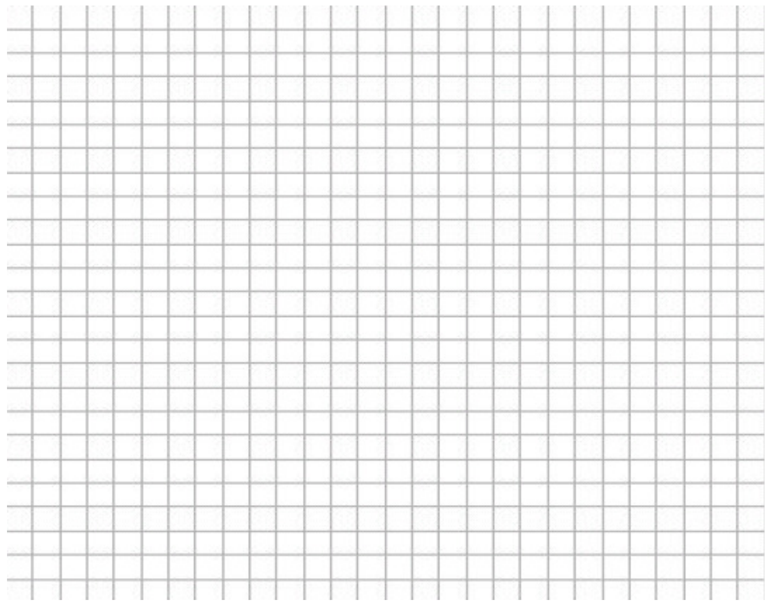
Warm-up:

1. Consider the parametric equation $x = \sqrt[3]{t}$ and $y = 1 - t$.
    a. Graph the parametric equation, indicating the orientation.



    b. Eliminate the parameter.

    c. Evaluate $\dfrac{dy}{dx}$ using your result from part b.

d. Now evaluate the following derivatives using the original parametric equations, $x = \sqrt[3]{t}$ and $y = 1 - t$.

i. $\dfrac{dx}{dt}$

ii. $\dfrac{dy}{dt}$

e. Now evaluate $\dfrac{dy}{dx}$ using your results from part d.

PARAMETRIC FORM OF THE DERIVATIVE

If a smooth curve *C* is given by the equations $x = f(t)$ and $y = g(t)$, then the slope at *C* at (x, y) is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

2. Find $\frac{dy}{dx}$ for the curve given by $x = \cos t$ and $y = \sin t$.

a. Now evaluate the second derivative, that is $\frac{d}{dx}\left[\frac{dy}{dx}\right]$..

HIGHER ORDER DERIVATIVES OF PARAMETRIC EQUATIONS

If a smooth curve *C* is given by the equations $x = f(t)$ and $y = g(t)$, then the slope at *C* at (x, y) is

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

In general we have,

$$\frac{d^ny}{dx^n} = \frac{\frac{d}{dt}\left[\frac{d^{n-1}y}{dx^{n-1}}\right]}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

3. Determine the *t* intervals on which the curve is concave downward or concave upward for the curve given by $x = t + 1$ and $y = t^2 + 3t$.

ARC LENGTH IN PARAMETRIC FORM

If a smooth curve *C* is given by the equations $x = f(t)$ and $y = g(t)$, such that *C* does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of *C* over the interval is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_a^b \sqrt{\left[f'(t)\right]^2 + \left[g'(t)\right]^2}\,dt.$$

NOTE: When applying the arc length formula to a curve, be sure that the curve is traced only once on the interval of integration.

4. Find the arc length of the curve given by

$x = \arcsin t$ and $y = \ln\left(1 - t^2\right)$ on the interval $0 \le \mathrm{t} \le \frac{1}{2}$.

AREA OF A SURFACE OF REVOLUTION

If a smooth curve *C* is given by the equations $x = f(t)$ and $y = g(t)$, does not cross itself on the interval $a \le t \le b$, then the area *S* of the surface of revolution formed by revolving *C* about the coordinate axes is given by the following.

1. $S = 2\pi \int_a^b g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ Revolution about the $x-$axis: $g(t) \ge 0$
2. $S = 2\pi \int_a^b f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ Revolution about the $y-$axis: $f(t) \ge 0$

5. Find the area of the surface generated by revolving the curve below about the *y*- axis.

$$x = \frac{1}{3}t^3 \text{ and } y = t + 1 \text{ on the interval } 1 \le t \le 2$$