MATH 251/GRACEY
WORKSHEET/5.8

HYPERBOLIC FUNCTIONS, INVERSE HYPERBOLIC FUNCTIONS AND IDENTITIES

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\tanh x = \dfrac{\sinh x}{\cosh x}$, $\cosh x \neq 0$

$\operatorname{csch} x = \dfrac{1}{\sinh x}$, $\sinh x \neq 0$

$\operatorname{sech} x = \dfrac{1}{\cosh x}$, $\cosh x \neq 0$

$\coth x = \dfrac{\cosh x}{\sinh x}$, $\sinh x \neq 0$

$\cosh^2 x - \sinh^2 x = 1$

$\tanh^2 x + \operatorname{sech}^2 x = 1$

$\coth^2 x - \operatorname{csch}^2 x = 1$

$\sinh^2 x = \dfrac{-1 + \cosh 2x}{2}$

$\cosh^2 x = \dfrac{1 + \cosh 2x}{2}$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

<u>Function</u>	<u>Domain</u>
$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$	$(-\infty, \infty)$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$	$[1, \infty)$
$\tanh^{-1} x = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}$	$(-1, 1)$
$\coth^{-1} x = \dfrac{1}{2}\ln\dfrac{x+1}{x-1}$	$(-\infty, -1)\cup(1, \infty)$
$\operatorname{csch}^{-1} x = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	$(-\infty, 0)\cup(0, \infty)$
$\operatorname{sech}^{-1} x = \ln\dfrac{1+\sqrt{1-x^2}}{x}$	$[0, 1)$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

1. Evaluate the expression without using a calculator.

a. $\sinh(-1)$

b. $\tanh\left(\frac{1}{2}\right)$

c. $\operatorname{sech}(-3)$

d. $\coth^{-1}(1)$

e. $\cosh^{-1}(5)$

f. $\tanh^{-1}(0)$

2. Verify the identity.

$$\cosh(x-y)=\cosh x\cosh y-\sinh x\sinh y$$

DERIVATIVES AND INTEGRALS OF HYPERBOLIC FUNCTIONS AND INVERSE HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}[\sinh u] = (\cosh u)u' \qquad \frac{d}{dx}[\cosh u] = (\sinh u)u'$$

$$\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u' \qquad \frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$$

$$\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u' \qquad \frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$$

$$\frac{d}{dx}\left[\sinh^{-1} u\right] = \frac{u'}{\sqrt{u^2+1}} \qquad \frac{d}{dx}\left[\cosh^{-1} u\right] = \frac{u'}{\sqrt{u^2-1}}$$

$$\frac{d}{dx}\left[\tanh^{-1} u\right] = \frac{u'}{1-u^2} \qquad \frac{d}{dx}\left[\operatorname{sech}^{-1} u\right] = \frac{-u'}{u\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left[\coth^{-1} u\right] = \frac{u'}{1-u^2} \qquad \frac{d}{dx}\left[\operatorname{csch}^{-1} u\right] = \frac{-u'}{|u|\sqrt{1+u^2}}$$

3. Find the derivative of the function. **Write your result as a single trigonometric expression**.

a. $h(x) = x\sinh(x^2)$

b. $f(x) = x - \coth(x)$

c. $g(t) = \cosh^{-1}\left(\frac{t}{4}\right)$

d. $f(x) = \left(\operatorname{sech}^{-1}(-x)\right)^2$

$\int \cosh u\,du = \sinh u + C$

$\int \sinh u\,du = \cosh u + C$

$\int \operatorname{sech}^2 u\,du = \tanh u + C$

$\int \operatorname{csch}^2 u\,du = -\coth u + C$

$\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

$\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln\left|\frac{a+u}{a-u}\right| + C$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$

4. Find the integral.

a. $\int 5x \operatorname{sech}^2 x^2 dx$

b. $\int \frac{dx}{16 - 4x^2}$

c. $\int \frac{3}{\sqrt{x}\sqrt{9+x}}dx$

d. $\int \frac{dx}{(x+2)\sqrt{x^2+4x+8}}$

e. $\int \frac{1}{1-4x-2x^2}dx$

f. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$