MATH 251/GRACEY NAME____________________________________
WORKSHEET/CHAPTER 7.4-7.5

ARC LENGTH AND AREA OF A SURFACE OF REVOLUTION

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a,b]$. The arc length of f between a and b is

$$s = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx,\ y \text{ is a function of } x,$$

If $x = g(y)$ on the interval $[c,d]$, then the arc length of g between c and d is

$$s = \int_c^d \sqrt{1+\left[g'(y)\right]^2}\,dy,\ x \text{ is a function of } y$$

1. Find the arc length of the graph of the function $y = \frac{3}{2}x^{2/3}$ over the interval $[1,4]$.

2. Find the arc length of the graph of the function $y = \frac{x^3}{6} + \frac{1}{2x}$ over the interval $\left[\frac{1}{2}, 2\right]$.

Let $y = f(x)$ have a continuous derivative on the interval $[a,b]$. The area S of the surface of revolution formed by revolving the graph of about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx,$$

y is a function of x, where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c,d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}\, dy,$$

x is a function of y, where $r(y)$ is the distance between the graph of g and the axis of revolution.

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3. Find the **area of the surface** formed by revolving the graph of $f(x) = x^2$ on the interval $\left[0, \sqrt{2}\right]$ about the y-axis.

Definition of Work Done by a Constant Force

If an object is moved a distance *D* in the direction of an applied constant force *F* then the **work** *W* done by the force is defined as $W = FD$.

4. Determine the work done in lifting a 100-pound bag of sugar 10 feet.

Definition of Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** *W* done by the force as the object is moved from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)dx.$$

Hook's Law: The force *F* required to compress or stretch a spring is proportional to the distance *d* that the spring is compressed or stretched from its original length.

$$F = kd$$

Newton's Law of Universal Gravitation: The force *F* of the attraction between two particles of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance *d* between the two particles.

$$F = k\frac{m_1 m_2}{d^2}$$

If m_1 and m_2 are given in grams and *d* is given in centimeters, *F* will be in dynes for a value of 6.670×10^{-8} cubic centimeter per gram-second squared.

Coulomb's Law: The force between two charges q_1 and q_2 in a vacuum is proportional to the product of the charges and inversely proportional to the square of the distance *d* between the two charges.

$$F = k\frac{q_1 q_2}{d^2}$$

If q_1 and q_2 are given in electrostatic units and *d* is given in centimeters, *F* will be in dynes for a value of $k = 1$.

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5. A force of 5 pounds compresses a 15-inch spring a total of 4 inches. How much work is done in compressing the spring 7 inches?

6. Neglecting air resistance and the weight of the propellant, determine the work done in propelling a 5-ton satellite to a height of 100 miles above Earth. Use 4000 miles as the radius of the Earth.