

BASIC INTEGRATION RULES ($a > 0$)

1. $\int kf(u) du = k \int f(u) du$

2. $\int [f(u) \pm g(u)] du =$
 $\int f(u) du \pm \int g(u) du$

3. $\int du = u + C$

4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

5. $\int \frac{du}{u} = \ln|u| + C$

6. $\int e^u du = e^u + C$

7. $\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$

8. $\int \sin u du = -\cos u + C$

9. $\int \cos u du = \sin u + C$

10. $\int \tan u du = -\ln|\cos u| + C$

11. $\int \cot u du = \ln|\sin u| + C$

12. $\int \sec u du = \ln|\sec u + \tan u| + C$

13. $\int \csc u du = -\ln|\csc u + \cot u| + C$

14. $\int \sec^2 u du = \tan u + C$

15. $\int \csc^2 u du = -\cot u + C$

16. $\int \sec u \tan u du = \sec u + C$

17. $\int \csc u \cot u du = -\csc u + C$

18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arc sec} \frac{|u|}{a} + C$

PROCEDURES FOR FITTING INTEGRANDS TO BASIC RULES

TECHNIQUE	EXAMPLE
Expand (numerator)	$(x^2 + 8)^2 = x^4 + 16x^2 + 64$
Separate numerator	$\frac{x-3}{1+x^2} = \frac{x}{1+x^2} - \frac{3}{1+x^2}$
Complete the square	$\frac{1}{\sqrt{4x-x^2}} = \frac{1}{\sqrt{4-(x-2)^2}}$
Divide improper rational function	$\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$
Add and subtract terms in numerator	$\frac{2x}{x^2+6x+9} = \frac{2x+6-6}{x^2+6x+9} = \frac{2x+6}{x^2+6x+9} - \frac{6}{(x+3)^2}$
Use trigonometric identities	$\cos^2 x = \frac{1+\cos 2x}{2}$
Multiply and divide by Pythagorean conjugate	$\begin{aligned} \frac{1}{1+\cos x} &= \frac{1}{1+\cos x} \left(\frac{1-\cos x}{1-\cos x} \right) \\ &= \frac{1-\cos x}{1-\cos^2 x} \\ &= \frac{1-\cos x}{\sin^2 x} \\ &= \csc x - \cot x \end{aligned}$

1. $\int \frac{1-\sin 2\theta}{\cos 2\theta} d\theta$

$$2. \int \frac{e^{1/t}}{t^2} dt$$

$$3. \int \frac{2}{(t-9)^2} dt$$

$$4. \int \tan x [\ln(\cos x)] dx$$