MATH 251/GRACEY NAME ______________________________
WORKSHEET/8.3

TRIGONOMETRIC INTEGRALS

We will study techniques for evaluating integral of the form

$$\int \sin^m x \cos^n x\,dx \quad \text{and} \quad \int \sec^m x \tan^n x\,dx$$

GUIDELINES FOR EVALUATING INTEGRALS INVOLVING SINE AND COSINE

1. If the power of the **sine** is **odd and positive**, save one sine factor and convert the remaining factors to cosines. Then expand and integrate.

$$\int \sin^{\overbrace{2k+1}^{\text{odd}}} x \cos^n x\,dx = \int \overbrace{\left(\sin^2 x\right)^k}^{\text{convert to cosines using pythagorean identity}} \cos^n x \overbrace{\sin^1 x\,dx}^{\text{save for } du}$$

$$= \int \left(1-\cos^2 x\right)^k \cos^n x \sin x\,dx$$

2. If the power of the **cosine** is **odd and positive**, save one cosine factor and convert the remaining factors to sines. Then expand and integrate.

$$\int \sin^m x \cos^{\overbrace{2k+1}^{\text{odd}}} x\,dx = \int \sin^m x \overbrace{\left(\cos^2 x\right)^k}^{\text{convert to sines using pythagorean identity}} \overbrace{\cos^1 x\,dx}^{\text{save for } du}$$

$$= \int \sin^m x \left(1-\sin^2 x\right)^k \cos x\,dx$$

3. If the powers of **both** the sine and cosine are **even and nonnegative**, make repeated use of the identities

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

To convert the integrand to odd powers of the cosine. Then proceed as in guideline 2.

GUIDELINES FOR EVALUATING INTEGRALS INVOLVING SECANT AND TANGENT

1. If the power of the **secant** is **even and positive**, save one secant-squared factor and convert the remaining factors to tangents. Then expand and integrate.

$$\int \sec^{\overbrace{2k}^{\text{even}}} x \tan^n x\,dx = \int \overbrace{\left(\sec^2 x\right)^{k-1}}^{\text{convert to tangents using pythagorean identity}} \tan^n x \overbrace{\sec^2 x\,dx}^{\text{save for } du}$$

$$= \int \left(1+\tan^2 x\right)^{k-1} \tan^n x \sec^2 x\,dx$$

2. If the power of the **tangent** is **odd and positive**, save one secant-tangent factor and convert the remaining factors to secants. Then expand and integrate.

$$\int \sec^m x \tan^{\overbrace{2k+1}^{\text{odd}}} x\,dx = \int \overbrace{\sec^{m-1} x \left(\tan^2 x\right)^k}^{\text{convert to secants using pythagorean identity}} \overbrace{\sec^1 x \tan x\,dx}^{\text{save for } du}$$

$$= \int \sec^m x \left(\sec^2 x - 1\right)^k \sec x \tan x\,dx$$

3. If there are **no secant factors** and the power of the **tangent** is **even and positive**, convert a tangent-squared factor to a secant-squared factor, then expand and repeat, if necessary.

$$\int \tan^n x\,dx = \int \tan^{n-2} x \overbrace{\left(\tan^2 x\right)}^{\text{convert to secants using pythagorean identity}} dx$$

$$= \int \tan^{n-2} x \left(\sec^2 x - 1\right) dx$$

4. If the integral is of the form $\int \sec^m x dx$, where *m* is **odd and positive**, use **integration by parts**!
5. If none of these techniques work, try converting to sines and cosines.

INTEGRALS INVOLVING SINE-COSINE PRODUCTS WITH DIFFERENT ANGLES

$$\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos\left[(m-n)x\right] - \cos\left[(m+n)x\right]\right)$$

$$\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin\left[(m-n)x\right] + \sin\left[(m+n)x\right]\right)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos\left[(m-n)x\right] + \cos\left[(m+n)x\right]\right)$$

$$\cos(mx)\sin(nx) = \frac{1}{2}\left(\sin\left[(m-n)x\right] - \sin\left[(m+n)x\right]\right)$$

1. Integrate.

   a. $\int \cos^7 x dx$

b. $\int \sin^4 3x dx$

c. $\int \sec^5 x dx$

d. $\int \tan^5 2x \sec 2x dx$

e. $\int \tan^4 x \sec^2 x dx$

f. $\int \sin(3x)\cos(2x)\,dx$

g. $\int \tan^6\left(\frac{x}{2}\right)dx$