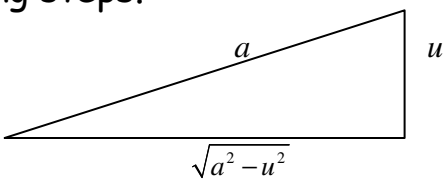
MATH 251/GRACEY NAME_____
WORKSHEET/8.4

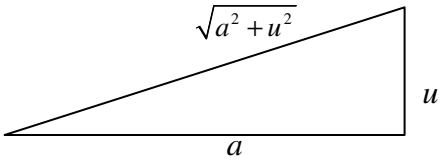
TRIGONOMETRIC SUBSTITUTION ($a>0$)

Let $f(c)=0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.

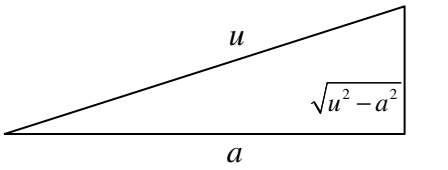
1. For integrals involving $\sqrt{a^2-u^2}$, let $u=a\sin\theta$.



2. For integrals involving $\sqrt{a^2+u^2}$, let $u=a\tan\theta$.



3. For integrals involving $\sqrt{u^2-a^2}$, let $u=a\sec\theta$.



Then $\sqrt{u^2-a^2}=\pm a\tan\theta$, where $0\le\theta<\frac{\pi}{2}$ or $\frac{\pi}{2}<\theta\le\pi$. Use the positive value if $u>a$ and the negative value if $u<-a$.

1. Find the integral.

a. $\displaystyle\int\frac{t}{\left(1-t^2\right)^{3/2}}\,dt$

b. $\int \frac{1}{\sqrt{x^2-9}}\,dx$

c. $\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$

d. $\int e^{x}\sqrt{1-e^{2x}}\,dx$

e. $\int \frac{x^{3}+x+1}{x^{4}+2x^{2}+1}\,dx$