MATH 251/GRACEY NAME______________________

WORKSHEET/8.8

IMPROPER INTEGRALS

Definition of Improper Integrals with Infinite Integration Limits

1. If f is continuous on the interval $[a,\infty)$, then

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

2. If f is continuous on the interval $(-\infty,b]$, then

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

3. If f is continuous on the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx,\ c \text{ is any real number.}$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.

1. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

a. $\int_0^\infty (x-1)e^{-x}\,dx$

b. $\int_1^{\infty} \frac{5}{x}\,dx$

Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval $[a,b)$ and has an infinite discontinuity at *b*, then $\int_a^b f(x)\,dx = \lim_{c\to b^-} \int_a^c f(x)\,dx$
2. If f is continuous on the interval $(a,b]$ and has an infinite discontinuity at *a*, then $\int_c^b f(x)\,dx = \lim_{c\to a^+} \int_a^c f(x)\,dx$
3. If f is continuous on the interval $[a,b]$, except for some *c* in (a,b) at which *f* has an infinite discontinuity, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.

2. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

a. $\int_{-1}^{2} \frac{dx}{x^3}$

b. $\int_{0}^{6} \frac{4}{\sqrt{6-x}}\,dx$

Theorem: A Special Type of Improper Integral

$$\int_1^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \le 1 \end{cases}$$

3. Evaluate the definite integral.

a. $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$

b. $\int_1^{\infty} \frac{5}{\sqrt[5]{x^6}} dx$

c. $\int_0^{2} (2-t)\sqrt{t}\, dt$