Theorem 9.12 Direct Comparison Test

Let $0 < a_n \leq b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Steps for using the Direct Comparison Test.

1. Decide if you think the series will converge or diverge. Then use a **similar series** for which it is easy to establish convergence or divergence which has terms which are
   a. **Greater** for all *n* in a term by term comparison with the given series if you want to prove **convergence** and
   b. **Lesser** for all *n* in a term by term comparison with the given series if you want to prove **divergence**.
2. Show that the **similar series** converges or diverges.
3. State your conclusion **in words** stating the reason for convergence or divergence.

1. Use the Direct Comparison Test to determine the convergence or divergence of the series.

   a. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

b. $\sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$

c. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n} - 1}$

Theorem 9.12 Limit Comparison Test

$a_n > 0,\ b_n > 0$, and

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$$

where L is finite and positive. Then the two series either both converge or both diverge.

Steps for using the Limit Comparison Test.

1. Decide if you think the series will converge or diverge. Then use a **similar series** for which it is easy to establish convergence or divergence.
2. Evaluate the limit of the ratio of the **sequence** in the given series to the **sequence** in the similar series.
3. State your conclusion **in words** stating the reason for convergence or divergence.

2. Use the Limit Comparison Test to determine the convergence or divergence series.

a. $$\sum_{n=1}^{\infty}\frac{2}{3^n-1}$$

b. $$\sum_{n=1}^{\infty} \frac{1}{n\left(n^2-1\right)}$$

c. $$\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$$