*n*th Taylor Polynomial

If *f* has *n* derivatives at *c*, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the *n*th Taylor Polynomial for *f* at *c*.

*n*th Maclaurin Polynomial (special case of the *n*th Taylor Polynomial for *f* at 0)

If *f* has *n* derivatives at *c*, then the polynomial

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$ is also called the *n*th Maclaurin Polynomial for *f*.

1. Find the Maclaurin polynomial of degree *n* for the function.

   a. $f(x) = e^{-x}, \quad n = 5$

   b. $f(x) = \sin \pi x, \quad n = 3$

2. Find the nth Taylor polynomial centered at c.

a. $f(x)=\dfrac{2}{x^2}, \quad n=4, \quad c=2$

b. $f(x)=x^2\cos x, \quad n=2, \quad c=\pi$

i. Approximate the function at $f\left(\dfrac{7\pi}{8}\right)$ using the result from 2b.