**Definition of Power series**

If $x$ is a variable, then the infinite series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_n x^n + \cdots$$ is called a **power series**.

More generally, an infinite series of the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + a_3 (x-c)^3 + a_n (x-c)^n + \cdots$$

is called a **power series centered at *c***, where $c$ is a constant.

1. State where the power series is centered.

a. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{n!}$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{(n-5)^n x^n}{\ln n}$

**Convergence of a Power Series**

For a power series centered at $c$, precisely **one** of the following is true.

1. The series converges only at $c$.
2. There exists a real number $R > 0$ such that the series converges absolutely for $|x-c| < R$ and diverges for $|x-c| > R$.
3. The series converges absolutely for all $x$.

The number $R$ is the **radius of convergence** of the power series. If the series converges only at $c$, the radius of convergence is $R = 0$. If the series converges for all $x$, the radius of convergence is $R = \infty$. The set of all values of $x$ for which the series converges is the **interval of convergence** of the power series.

2. Find the radius of convergence of the power series.

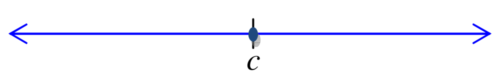
a. $\displaystyle\sum_{n=0}^{\infty}(2x)^n$

b. $\displaystyle\sum_{n=0}^{\infty}\frac{(2n)!x^{2n}}{n!}$

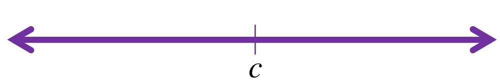
## Endpoint Convergence

Note that for a power series whose radius of convergence is a finite number *R*, each endpoint must be tested separately for convergence or divergence. The interval of convergence of a power series can take any one of the six forms below.

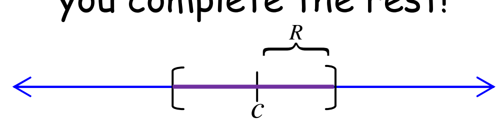
Radius: 0

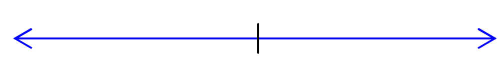


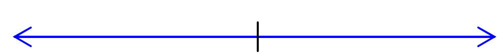

Radius: Infinity

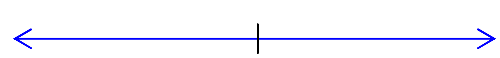



Radius: *R*—I showed you one possibility, now you complete the rest!







3. Find the interval of convergence of the power series. Be sure to include a check for endpoint convergence at the endpoints of the interval.

a. $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n$

b. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(n+1)(n+2)}$

c. $\sum_{n=1}^{\infty}\frac{(x-3)^{n-1}}{3^{n-1}}$