Operations with Power Series

1. $f(kx)=\sum_{n=0}^{\infty} a_n k^n x^n$

2. $f(x^N)=\sum_{n=0}^{\infty} a_n x^{nN}$

3. $f(x)\pm g(x)=\sum_{n=0}^{\infty}(a_n \pm b_n)x^n$

These operations can change the interval of convergence for the resulting series.

Recall that $\frac{a}{1-r}=\sum_{n=0}^{\infty} ar^n$ represents the sum of a convergent geometric series where $|r|<1$.

1. Find a geometric power series for the function, centered at 0.

   a. $f(x)=\frac{6}{7-x}$

   b. $f(x)=\frac{1}{1+x}$

2. Find a geometric power series for the function, centered at c, and determine the interval of convergence.

a. $f\left(x\right)=\dfrac{4}{5-x},\quad c=-2$

b. $f\left(x\right)=\dfrac{1}{2x-5},\quad c=2$

c. $f(x)=\frac{4}{4+x^2}, \quad c=0$

3. Use the power series $\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$ to determine a power series, centered at 0, for the function. Identify the interval of convergence.

a. $h(x)=\frac{x}{x^2-1}=\frac{1}{2(1-x)}-\frac{1}{2(1-x)}$

b. $f(x)=\ln\left(1-x^2\right)=\int \frac{1}{1+x}dx-\int \frac{1}{1-x}dx$

4. Use the power series $\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n,\ \ |x|<1$ to determine a power series. Identify the interval of convergence.

$f(x)=\frac{x}{(1-x)^2}$