MATH 251/GRACEY NAME______________________________

PRACTICE/CHAPTER 8

1. Find the integral.

a. $\int \frac{(\ln x)^2}{x} dx$

b. $\int \ln \sqrt{x^2 - 1} dx$

c. $\displaystyle\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

d. $\displaystyle\int \frac{1}{1-\cos\theta}\,d\theta$

e. $\displaystyle\int \frac{-12}{x^2\sqrt{4-x^2}}\,dx$

f. $\displaystyle\int \frac{x-28}{x^2-x-6}\,dx$

2. Evaluate the definite integral.

a. $\int_0^{\pi/4} \tan^3 x dx$

b. $\int_0^{2} e^{-x} \cos x dx$

c. $\int_0^e \ln x^2 dx$

d. $\int_1^\infty \frac{1}{x^2+5} dx$

3. For the following limits:

   a. Describe the type of indeterminate form (if any) that is obtained by direct substitution.

   b. Evaluate the limit, using L'Hôpital's Rule if necessary.

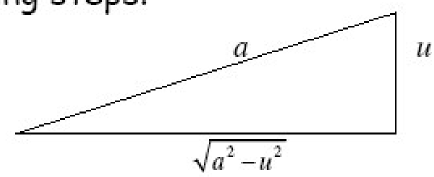
      i. $\lim_{x \to \infty} x \ln x$

      ii. $\lim_{x \to 0} \frac{\arcsin x}{x}$

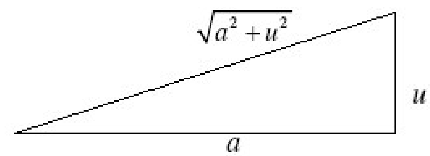
TRIGONOMETRIC SUBSTITUTION ($a > 0$)

Let $f(c) = 0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.

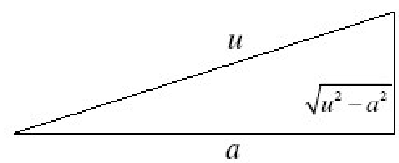
1. For integrals involving $\sqrt{a^2 - u^2}$, let $u = a\sin\theta$.



2. For integrals involving $\sqrt{a^2 + u^2}$, let $u = a\tan\theta$.



3. For integrals involving $\sqrt{u^2 - a^2}$, let $u = a\sec\theta$.



Then $\sqrt{u^2 - a^2} = \pm a\tan\theta$, where $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$. Use the positive value if $u > a$ and the negative value if $u < -a$.

Indeterminate Forms:

$$\frac{0}{0},\ \frac{\infty}{\infty},\ \infty - \infty,\ 0 \cdot \infty,\ 0^0,\ 1^\infty,\ \infty^0$$

Definition of Improper Integrals with Infinite Integration Limits

1. If f is continuous on the interval $[a,\infty)$, then $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$
2. If f is continuous on the interval $(-\infty,b]$, then

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

3. If f is continuous on the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx,\ c \text{ is any real number.}$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.

Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval $[a,b)$ and has an infinite discontinuity at *b*, then

$$\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$$

2. If f is continuous on the interval $(a,b]$ and has an infinite discontinuity at *a*, then

$$\int_c^b f(x)\,dx = \lim_{c\to a^+}\int_a^c f(x)\,dx$$

3. If f is continuous on the interval $[a,b]$, except for some *c* in (a,b) at which *f* has an infinite discontinuity, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.