

When you are done with your homework you should be able to...

- π Evaluate a Rational Expression.
- π Determine Undefined Values of a Rational Expression.
- π Simplify Rational Expressions.

WARM-UP:

1. Evaluate each of the following expressions at $x = -2$.

a. $\frac{x^3 + 5}{x^2}$

$$\frac{(-2)^3 + 5}{(-2)^2} = \frac{-8 + 5}{4} = \boxed{-\frac{3}{4}}$$

b. $x + \frac{5}{x^2}$

$$-2 + \frac{5}{(-2)^2} = -2 + \frac{5}{4} = -\frac{8}{4} + \frac{5}{4} = \boxed{-\frac{3}{4}}$$

2. Factor the following polynomials.

a. $x^2 + 7x + 6$

$$x^2 + 6x + x + 6$$

$$x(x+6) + 1(x+6)$$

$$(x+6)(x+1)$$

over product of coeff



overall sum or difference

b. $8x^3 + 27$

$$2x \cdot 2x \cdot 2x \quad 3 \cdot 3 \cdot 3$$

$$= (2x+3)(4x^2 - 2x \cdot 3 + 3^2)$$

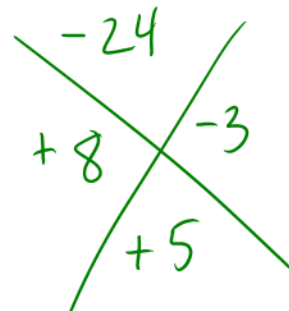
$$= (2x+3)(4x^2 - 6x + 9)$$

c. $6x^2 + 5x - 4$

$$6x^2 + 8x - 3x - 4$$

$$2x(3x+4) - 1(3x+4)$$

$$(3x+4)(2x-1)$$



3. Divide and simplify.

$$\frac{3x^2y^5}{105x^2y}$$

$$\frac{y^4}{35}$$

Definition: Rational Expression

A rational expression is the quotient of two polynomials.
Suppose p and q are polynomials. Write a rational expression using p and q .

$$\frac{p}{q}, q \neq 0$$

Make up 3 rational expressions and write them down below:

1. $\frac{2x+1}{3x-5}$

2. $\frac{2x}{8x+2}$

3. $\frac{5x-3}{2x+2}$

Is $\frac{2}{5}$ a rational expression?

Evaluate the following rational expressions at the given value:

1. $\frac{2a+25}{a^2-1}, a=3$

$$\frac{2(3)+25}{(3)^2-1} = \frac{31}{8}$$

2. $\frac{m^2-25}{(m-5)^2}, m=-2$

$$\frac{(-2)^2-25}{[(-2)-5]^2} = \frac{-21}{(-7)^2}$$

$$= -\frac{21}{49}$$

$$= \frac{21}{49}$$

$$= \frac{1 \cdot 3}{7 \cdot 7}$$

$$= \boxed{\frac{3}{7}}$$

What are undefined values of a rational expression?

A rational expression is **undefined** for those values of the variable(s) that make the denominator zero.

Determine the value(s) for which the following rational expressions are undefined:

1. $\frac{5}{x}$

$x=0$

2. $\frac{5}{x-3}$

$x-3=0$
 $x=3$

3. $\frac{5}{2x-3}$

$2x-3=0$
 $2x=3$
 $x=3/2$

4. $\frac{x-5}{x-5}$

$x-5=0$
 $x=5$

5. $\frac{5x}{x^2-x-6}$

$x^2-x-6=0$
 $(x-3)(x+2)=0$
 $x-3=0$ or $x+2=0$
 $x=3$ or $x=-2$

Simplifying Rational Expressions

If p , q , and r are polynomials, then $\frac{p \cdot r}{q \cdot r} = \frac{p}{q}$ if $r \neq 0$ and $q \neq 0$.

Make up your own p , q , and r , write a rational expression, and simplify.

Here's mine!

$$\frac{x(x+2)}{(x+2)} = \frac{x}{1} = x$$

$$\frac{2x(x+5)}{(2x+3)(x+5)} = \frac{2x}{2x+3}$$

$\frac{3}{10} = \frac{3}{3+7} \stackrel{?}{=} \frac{3}{3} + \frac{3}{7}$ Heck No!

Steps to Simplify a Rational Expression

Step 1: Completely factor the numerator and denominator of the rational expression.

Step 2: Divide out common factors.

Simplify the following rational expressions:

$$1. \frac{27}{387} = \frac{\cancel{9}(3)}{\cancel{9}(43)} = \boxed{\frac{3}{43}}$$

$$2. \frac{x^2 - 5x}{3x^2 - 16x + 5} = \frac{x \cancel{(x-5)}}{\cancel{(x-5)}(3x-1)} = \boxed{\frac{x}{3x-1}}$$

$$\begin{array}{r} 15 \\ -15 \quad -1 \\ -16 \end{array}$$

$$3x^2 - 15x - x + 5$$

$$3x(x-5) - 1(x-5)$$

$$3. \frac{1-x}{x^2-1}$$

$$4. \frac{x^2 - 9}{x^3 - 27}$$

$$5. \frac{\underbrace{n^3 + 3n^2}_{n^2(n+3)} - \underbrace{8n - 24}_{8(n-3)}}{\underbrace{n^3 - 4n^2 + 3n - 12}_{n^2(n-4) + 3(n-4)}} = \frac{n^2(n+3) - 8(n-3)}{n^2(n-4) + 3(n-4)}$$

$$= \frac{(n+3)(n^2 - 8)}{(n-4)(n^2 + 3)}$$

Applications

A formula to calculate the body mass index (BMI) when the weight w is in pounds, and the height h is in inches, is given by $\text{BMI} = \frac{705w}{h^2}$. What is the BMI of a person weighing 120 pounds who is 5 feet tall?