On Friday, 9/11/09, we held our second workshop of the series entitled Math Teacher Talk (MT²). Our discussion focused on order of operations.

ORDER OF OPERATIONS

1. *How can we explain why one works a problem from left to right when in the same hierarchy?* During my basic college mathematics course (MATH 20), I was asked this question. As I shared with the group, I had no trouble explaining the reason for this when dealing with multiplication and division, as the students had already learned about reciprocals of whole numbers. I would simply change the division parts into multiplying by the reciprocal, and the students appeared to understand. The problem lies in the addition and subtraction hierarchy. These students are working only with whole numbers, which do not include negative numbers. Therefore, changing the subtraction into adding the opposite was not an option. Here were some ideas:
    - Remind the students that we learned about the commutative property of addition. Then let them know that subtraction is not commutative.
    - We read from left to right and this extends to math, too!

2. *Why isn't* $8 \div \frac{2}{3} = 8 \div 2 \div 3$? This question generated a very interesting conversation. When we adjusted the expression by adding a variable, $8 \div \frac{x+2}{3}$ it was very clear that there is an implied parenthesis around the fraction which would result in $8 \div \left(\frac{2}{3}\right) = 8 \cdot \left(\frac{3}{2}\right)$. We came to the following recommendations:
    - Always put parenthesis around the fraction
    - Be careful to write the fraction bar at the same level as the division sign

3. *Why isn't* $\frac{x}{x+3} = \frac{1}{4}$? We came up with the following recommendations:
    - Illustrate why this is not true by plugging in actual numbers

- Use the term “factors” and put parenthesis around all factors $\frac{(x)}{(x+3)}$
- Use a Star Trek analogy
    - The parenthesis are force fields
- Do not use the word “cancel”. Instead say “divide out these common factors” or “these terms zero out”

4. *Mistake on the MATH 35 test bank?* The following item is on the MATH 35 test bank: *The product of -3 and 4 plus a number.”* The answer (according to the test bank) is $-3(4+n)$. Since the author did not place a comma after the negative three, the answer should be $-3 \cdot 4+n$. *Moral of the story—depend on the key you work out yourself!*

5. *Why is -3^2 still negative?* Here are some suggestions:
    - Use the definition of exponentiation and the idea that a negative number is the number multiplied by negative 1
        - $-3^2=(-1)(3 \cdot 3)$

6. *Why is minus a minus plus?* Here are some suggestions:
    - Use the different colored chips to illustrate. Assign one color to positive and the other color to negative
    - Instead of saying “subtract”, say “add the opposite”

7. *Suggested Activities.* Here are some suggestions:
    - Play the order of operations card game!
        - Take out all the face cards
            - The aces are the ones
        - Deal out five cards to each player
        - Put the deck of cards face down, then turn up one card
        - The object of the game is to use the cards and order of operations to obtain the number on the turned up card

    - Give the students 5 numbers. Specify the operations which are allowed. Write down the desired result.
    - Surf the web for games and worksheets!

GENERAL WORDS OF WISDOM

1. Elicit information from the students instead of imparting all the information yourself.
2. Have patience! Never give up on your students.