Math 251 Practice for the Final Exam

Chapter 7: Know the derivative and related integral formulas involving the inverse trigonometric functions, and be able to evaluate them by hand.

1. Find the equation of the line tangent to the curve $f(x) = \arctan\left(\frac{x}{2}\right)$ when x = $2\sqrt{3}$.

2. Evaluate the integral: $\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}}\,dx$

Chapter 7: Know how to compute arc length and surface area. Know how to compute mass, and also *work*, including Hooke's Law and lifting rope/chain.

3. Find the length of the curve $f(x) = \ln(\sin x)$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$. Integrate by hand.

4. Find the area of the surface formed by revolving the graph of $f(x) = e^{2x}$ on the interval [0, 2] about the y – axis. You may use your calculator to integrate.

5. A spring has a natural length of 6 inches. A force of 10 pounds compresses the spring 2 inches from it's natural length. Find the **work** done in stretching the spring from 7 inches to 10 inches.

Chapter 8: Integration Techniques. Know your formulas, and know your techniques!

Techniques to know:

- Known Formulas
- Integration by Substitution
- Integration by Parts
- Integration by Partial Fractions
- Trigonometric Identities
- Trigonometric Substitution (Triangle Method)
- Strategies (long division, splitting the numerator, etc.)
- Improper integral techniques, including L'Hopital's rule if appropriate

-examples- Integrate.

6. $\int \frac{1}{4+9x^2}\,dx$

7. $\int \frac{1}{4-9x^2}\,dx$

8. $\int x^2 e^{2x}\,dx$

9. $\int \arcsin x\,dx$

10. $\int \sqrt{1-x^2}\,dx$

11. $\int \sin^2(2x)\,dx$

DEFINITE INTEGRALS:

12. $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

*13. $\int_{1}^{2} \frac{x-2}{x-1}\,dx$ *Careful – this is an improper integral!

Chapter 9: Know how to solve separable differential equations, and how they relate to exponential growth/decay.

14. Find the particular solution, $y = f(x)$, to the differential equation $\frac{dy}{dx} = -x^2y + y$, given $f(0) = 3$.

15. A lake can support a maximum population of 2000 fish. The number of fish in the lake grows ***at a rate directly proportional to the difference between the maximum population and the current population.*** Initially (time t = 0 years) there are 50 fish in the lake. After 2 years, there are 80 fish.

a. Write and solve a differential equation to determine the population of fish in the lake at any time t.

b. How many fish will be in the lake at time t = 5 years?

Chapter 10/11: Know and be able to apply your convergence tests, and be able to determine intervals of convergence. Know how to generate a power series via Taylor's formula or recognition as a geometric sum. Be able to manipulate a given power series (integrate, differentiate, perform a composition, etc.).

16. Determine whether the series converges or diverges: $\sum_{n=1}^{\infty} \frac{4\sqrt{n}}{n^2+3n+1}$

17. Determine the interval of convergence: $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n \cdot 4^n}$

18. Derive the 3rd degree Taylor polynomial for $f(x) = \sqrt[3]{x}$ centered at c = 8.

19. Use infinite series to evaluate the integral: $\int_0^{0.6} \sin(x^2)dx$ with an error of no more than 10^{-4}. Clearly show all steps, and explain how you determined the number of terms necessary for this approximation. Give your final approximation correct to 5 decimal places.

Chapter 12: Know how to work with equations in parametric and polar mode, including finding derivatives, arc length, and area.

20. Write the parametric equations for a line that passes through the points (4, -1) and (3, 5).

21. Convert the parametric equations to Cartesian, and identify the conic section represented.

$$x(t) = 4 + \cos t,\ y(t) = 2\sin t$$

22. Find the length of the curve represented by $x = \cos t - \sin t,\ y = \cos t + \sin t,\ 0 \le t \le \pi$. Integrate by hand.

23. Express the Cartesian point $(-1, -\sqrt{3})$ in Polar Coordinates in two different ways. Include a plot of the point in your work.

24. Convert the polar equation $r = 5\csc\theta$ to a Cartesian equation, and identify the shape (line, parabola, ellipse, circle, or hyperbola) represented.

25. Find the area of the region that lies within both curves: $r = 1 + \cos\theta,\ r = 3\cos\theta$. You may use your calculator to evaluate your integrals, and give your final answer correct to 3 decimal places.